Chapter 6: TIDES

Two kind of tides:

EQUILIBRIUM TIDE: formation of an equilibrium bulge. Dissipation of energy is caused by the time variation of the tidal bulge. Typical of bodies in small eccentricity orbits and large rocky component.

DYNAMICAL TIDE: the body is assumed to be an oscillator with a number of modes that are excited when the companion passes at the pericenter. The modes are damped when the companion is at the apoastron. Typical of systems with high eccentricities and with large amounts of gas (star, giant planets).

Effects of equilibrium tides of the planet on the satellite:

- Synchronization of the rotation rate to the orbital period

- Circularization of the satellite orbit

- Possible capture in spin-orbit resonance

Once a satellite has achieved synchronous rotation, tidal interaction does not stop but tends to circularize the satellite orbit.

$$L = I_s \Omega_s + \frac{m_p m_s}{(m_p + m_s)} a^2 n (1 - e^2)^{1/2}$$

The angular momentum in the satellite rotation is negligible respect to the orbital angular momentum due to the small satellite mass. So:

$$L_0^2 = \frac{m_p^2 m_s^2}{(m_p + m_s)^2} a^4 n^2 (1 - e^2)$$

The eccentricity can be calculated as:

$$e^2 = 1 - L_0^2 \frac{(m_p + m_s)^2}{(m_p m_s)^2} \frac{1}{a^4 n^2} = 1 + 2 \frac{E L_0^2}{G^2} \frac{(m_p + m_s)}{(m_p m_s)^3}$$

...remembering that ... $E = -\frac{G m_p m_s}{2a}$

$$\dot{e} = \frac{2 \dot{E} L_0^2}{2e} \frac{(m_p + m_s)}{G^2 (m_p m_s)^3} = -\frac{\dot{E}}{2eE}(1 - e^2) \approx -\frac{\dot{E}}{2eE}$$

$\dot{E} < 0 \quad \Rightarrow \quad \dot{e} < 0$

Tidal circularization of orbits!!

Evolution of eccentricity for a tidally sinchronized satellite with an eccentric orbit: radial and librational tide due to the motion of the planet respect to the satellite in a reference frame body-fixed to the satellite.

$$\frac{de}{dt} = \frac{63}{4} \frac{m_p}{m_s} \left(\frac{R}{a}\right)^5 \frac{n}{\mu_s Q}$$

Where u_s is the ratio between the elastic and gravitational force acting within the satellite.

Resonances can halt tidal circularization: The case of Io-Europa-Ganimede: 3-body resonance. $\lambda_{Io} - 3\,\lambda_{Eur} + 2\,\lambda_{Gan} \cong 180^{o}$

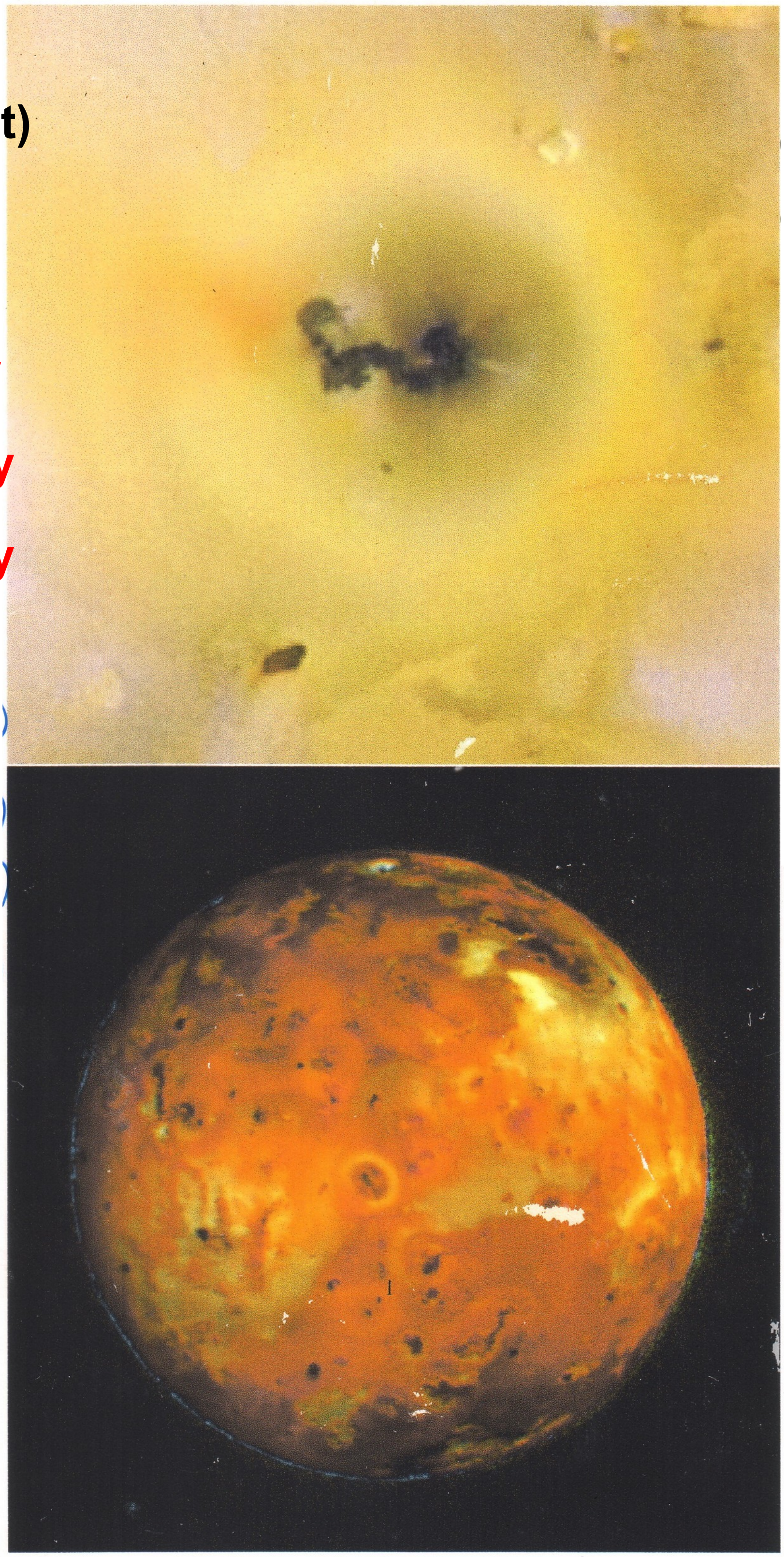
τ_e = -e/(de/dt)

$\tau_e \sim 6 \times 10^{6}$ y

$\tau_e \sim 3 \times 10^{8}$ y

$\tau_e \sim 3 \times 10^{9}$ y

Circularization timescale



a_{Io}=421600 km

a_{Eu}=670900 km

a_{Ga}=1070000 km

R_{Io} = 1821 km

R_{Eu} = 1565 km

R_{Ga} = 2634 km

e_{Io} = 0.0043

e_{Eu} = 0.01

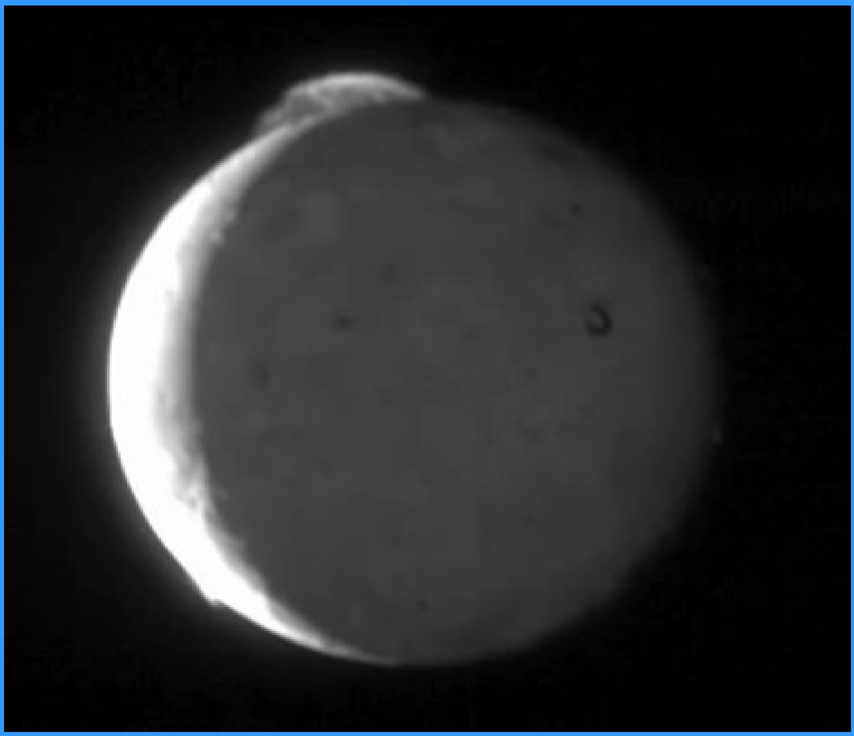
e_{Ga} = 0.0015

T_{Io}=1.77 d (Syn)

T_{Eu}=3.55 d (Syn)

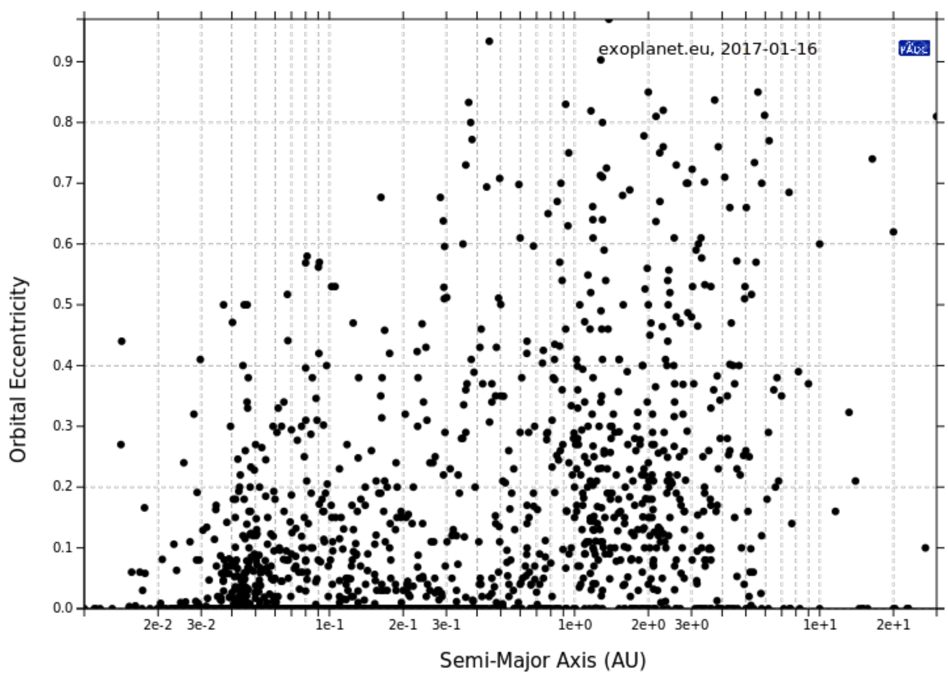
T_{Ga}=7.15 d (Syn)

T_{Ca}=16.6 d (Syn)



Io vulcanic activity seen by space probes.

Tidal interaction between planet and star to explain hot Jupiters. The semimajor axis is tidally circularized to twice the initial (and constant..) pericenter distance



$$L = \frac{m_p m_s}{m_p + m_s} \sqrt{\left(G M a \left(1 - e^2\right)\right)} = const$$

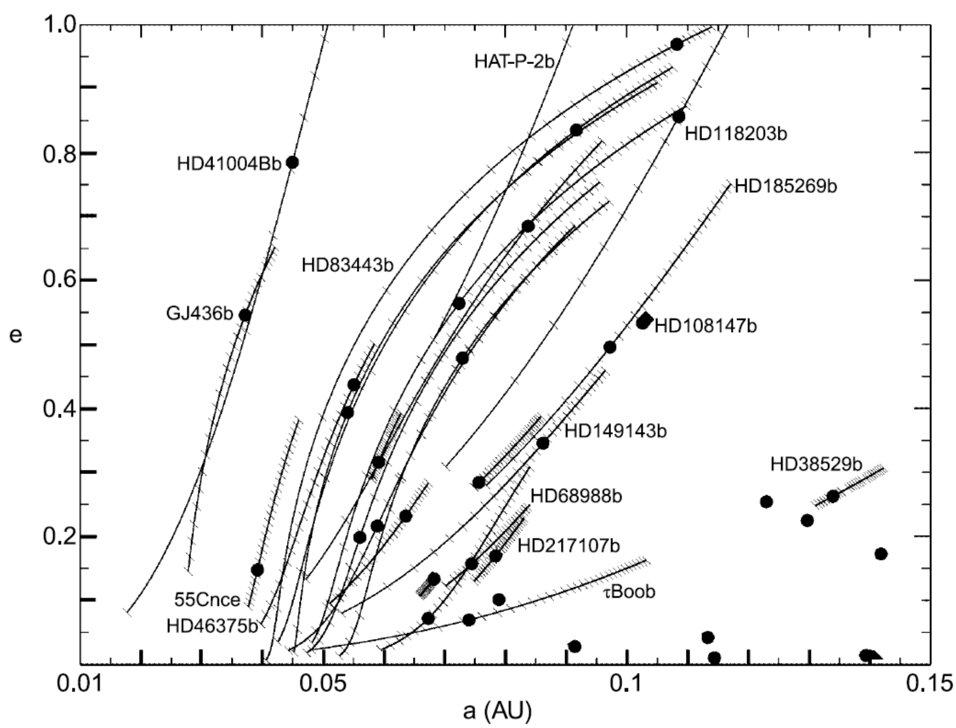
$$a\left(1-e^2\right) = a(1+e)\cdot(1-e) \;\Rightarrow\; \sim 2a\cdot(1-e)$$

If the initial eccentricity is about 0.9, then the orbit becomes very tight.

Tidal interaction between star and planet

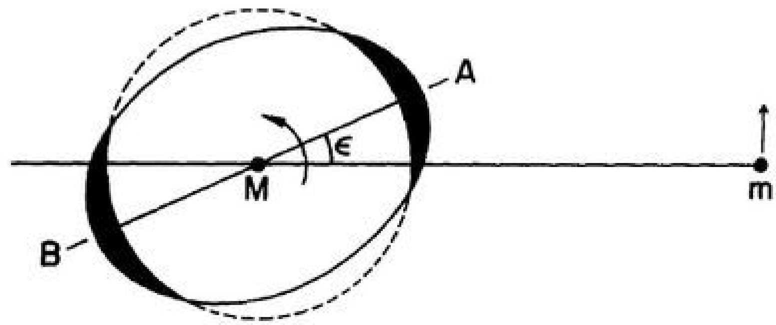
$$\frac{1}{e}\frac{de}{dt} = -\left[\frac{63}{4}(GM_*^3)^{1/2}\frac{R_p^5}{Q_pM_p} + \frac{171}{16}(G/M_*)^{1/2}\frac{R_*^5M_p}{Q_*}\right]a^{-13/2}, \tag{1}$$

$$\frac{1}{a}\frac{da}{dt} = -\left[\frac{63}{2}(GM_*^3)^{1/2}\frac{R_p^5}{Q_pM_p}e^2 + \frac{9}{2}(G/M_*)^{1/2}\frac{R_*^5M_p}{Q_*}\right]a^{-13/2}. \tag{2}$$



Backword numerical integration (Jackson 2008).

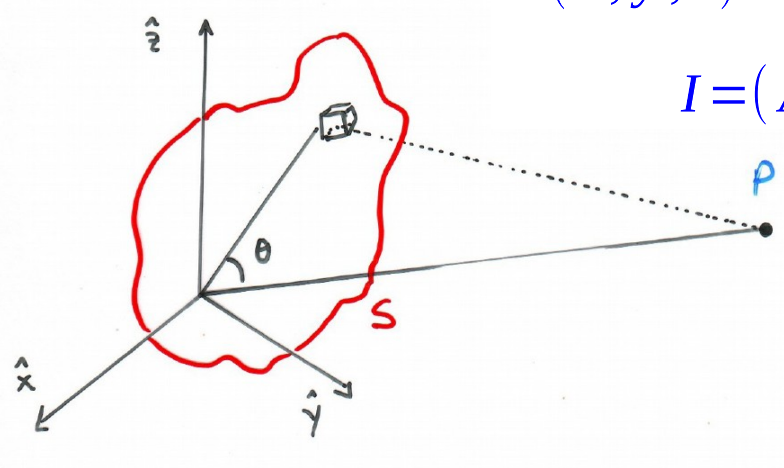
$Q_* \sim 3 \times 10^5$ $Q_p \sim 3 \times 10^6$



1) Spin-orbit resonance.

2) Resonance superposition and chaotic evolution

EVOLUTION OF THE SATELLITE SPIN



$$V(x,y,z) \simeq -\frac{G m_s}{r} - \frac{G(A+B+C-3I)}{2r^3}$$

$$I = (A x^2 + B y^2 + C z^2)/r^2$$

Mc Cullogh potential of an irregular body computed on on position P (x,y,z). A,B,C principal moments of inertia, body fixed reference frame

https://farside.ph.utexas.edu/teaching/celestial/Celestial/node73.html

$$F_x = -\frac{\partial V}{\partial x} = -\frac{G m_s x}{r^3} + G(B+C-2A)\frac{x}{r^5} + \ldots..$$

$$F_y = -\frac{\partial V}{\partial y} = -\frac{G m_s x}{r^3} + G(C+A-2B)\frac{y}{r^5} + \ldots..$$

$$F_z = -\frac{\partial V}{\partial z} = -\frac{G m_s z}{r^3} + G(A+B-2C)\frac{z}{r^5} + \ldots..$$

Force per unit of planet mass

$$N_x = (\vec{r} \times \vec{F}) = z F_y - y F_z = 3G(C-B)\frac{yz}{r^5}$$

$$N_y = 3G(A-C)\frac{zx}{r^5}$$

$$N_z = 3G(B-A)\frac{xy}{r^5}$$

The torque acts on body P, but for the action-reaction principle P acts with an equal torque (with opposite sign) on S (left equations)

See pg. 198 Murray & Dermott “Solar System Dynamics”

$$V=-\frac{G m_s}{r}-\frac{G}{2r^5}f\left(A,B,C;x,y,z\right)$$

$$f\left(A,B,C;x,y,z\right)=\left(B+C-2A\right)x^2+\left(C+A-2B\right)y^2+\left(A+B-2C\right)z^2$$

$$F_x=-\frac{\partial V}{\partial x}=-\frac{G m_s x}{r^3}+\frac{G\left(B+C-2A\right)x}{r^5}-\frac{5Gx}{2r^7}f\left(A,B,C;x,y,z\right)$$

$$F_y=-\frac{\partial V}{\partial y}=-\frac{G m_s y}{r^3}+\frac{G\left(A+C-2B\right)y}{r^5}-\frac{5Gy}{2r^7}f\left(A,B,C;x,y,z\right)$$

$$F_z=-\frac{\partial V}{\partial z}=-\frac{G m_s z}{r^3}+\frac{G\left(A+B-2C\right)z}{r^5}-\frac{5Gz}{2r^7}f\left(A,B,C;x,y,z\right)$$

$$N_x=zF_y-yF_z=-z\frac{G m_s y}{r^3}+y\frac{G m_s z}{r^3}-3G\left(C-B\right)\frac{yz}{r^5}+$$

$$-\frac{5Gzy}{2r^7}F\left(A,B,C;x,y,z\right)+\frac{5Gzy}{2r^7}F\left(A,B,C;x,y,z\right)$$

The first 2 and last 2 terms cancel out and then finally:

$$N_x=3G\left(C-B\right)\frac{yz}{r^5}$$

Equations for the time evolution of spin: Euler equations for rigid body

$$\frac{d\vec{J}}{dt}+\vec{\omega}\times\vec{J}=\vec{N}$$

$\vec{J}$ *angular momentum*
$\vec{N}$ *torque*

In a body-fixed reference frame.

$$A\dot{\omega}_x+(C-B)\omega_y\omega_z=N_x$$
$$B\dot{\omega}_y+(A-C)\omega_z\omega_x=N_y$$
$$C\dot{\omega}_z+(B-A)\omega_x\omega_y=N_z$$

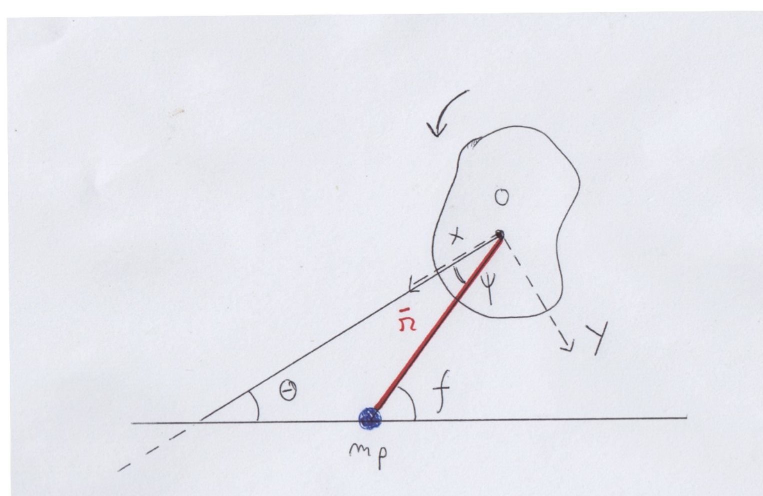
In the simplified scenario where J is perpendicular to the orbital plane of the satellite around the planet we have

$$C\dot{\omega}_z=N_z=3G(B-A)\frac{xy}{r^5}$$

x,y are the planet coordinates.

x,y body-fixed axes

$$\omega_z=\dot{\theta}$$
$$\frac{x}{r}=\cos(\psi)$$
$$\frac{y}{r}=\sin(\psi)$$



The torque was computed per unit mass of the planet. It has to be multiplied by m_p

$$C\ddot{\theta}-\frac{3}{2}Gm_p\frac{(B-A)}{r^3}\sin(2\psi)=0$$

$$2\cos(\psi)\sin(\psi)=\sin(2\psi)$$

Looking for a simpler form of the equation. Pendulum equation and resonances...

$$\psi = f - \theta \qquad \gamma = \theta - p\,M$$

Resonant angle: M is the mean anomaly, p is a rational number (2/3, 3/1 …..)

$$\ddot{\theta} = \ddot{\gamma} \;\Rightarrow\; \ddot{\gamma} + \frac{3}{2} n^2 \left(\frac{B-A}{C}\right)\left(\frac{a}{r}\right)^3 \sin(2\gamma + 2\,p\,M - 2f) = 0$$

$$\sin(-x) = -\sin(x)$$

1st STEP: *a/r* and *f* (true anomaly) are expanded as series in e and M

$$\left(\frac{a}{r}\right)^3 = 1 + 3e\cos(M) + \frac{3}{2}e^2(1 + 3\cos(2M)) + \ldots$$

$$\sin(f) = (1 - \frac{7}{8}e^2)\sin(M) + e\sin(2M) + \frac{9}{8}e^2\sin(3M) + \ldots$$

$$\cos(f) = (1 - \frac{9}{8})\cos(M) + e(\cos(2M) - 1) + \frac{9}{8}e^2\cos(3M) + \ldots$$

$$\sin(2\gamma + 2\,p\,M - 2f) \;\; is \;\; decomposed \;\; as$$
$$\sin(2\gamma)(\cos(2\,p\,M)\cos(2f) + \sin(2\,pM)\sin(2f)) + \ldots$$

...substituting, grouping etc...

$$\ddot{\gamma}+\frac{3}{2}\frac{B-A}{C}n^2((S_1+S_2)\sin(2\gamma)+(S_3-S_4)\cos(2\gamma))=0$$

The S_i are series in e and M and they depend on p

2nd STEP : Close to the resonance the critical argument evolve slowly $\dot{\gamma} \ll n$ The perturbing terms S_i can be averaged over a period of the satellite orbit.

$$\langle S_i \rangle = \frac{1}{2\pi}\int_0^{2\pi} S_i \, dM$$

After averaging of the S_i and their grouping, we get a single pendulum-like equation

$$\boxed{\ddot{\gamma}+\frac{3}{2}\frac{B-A}{C}n^2 H(p,e)\sin(2\gamma)=0}$$

$$H(1,e)=1-\frac{5}{2}e^2+\frac{13}{16}e^4$$ 1:1 synchronous

$$H(-1,e)=\frac{1}{24}e^4$$ 1:1 retrograde

$$H(\frac{1}{2},e)=-\frac{1}{2}e+\frac{1}{16}e^3$$ 2:1

$$H(\frac{3}{2},e)=\frac{7}{2}e-\frac{123}{16}e^3$$ 3:2

$$\ddot{\gamma} = -\frac{1}{2}\omega_0^2 \sin(2\gamma) \cdot (sign\ \ of\ \ H(p,e))$$

$$\omega_0 = n\left[3\left(\frac{B-A}{C}\right)|H(p,e)|\right]^{1/2}$$

3rd STEP : Tidal interaction slows down the satellite rotation

$$C\dot{\omega}_z = \langle N_M \rangle$$

$\langle N_M \rangle$ Tidal force momentum averaged over a period. It is a dissipative force -> it is < 0

$$\langle N_M \rangle = -D\left(\frac{a}{r}\right)^6 \cdot sign\left(\dot{\Omega}_s - \dot{n}\right) \qquad with \qquad D = \frac{3}{2}\frac{K_2}{Q_s}\frac{n^4}{G}R_s^5$$

$$\ddot{\gamma} = -\frac{1}{2}\omega_0^2 \sin(2\gamma) \cdot (sign\ \ of\ \ H(p,e)) + \frac{\langle N_M \rangle}{C}$$

$$\boxed{\left|\frac{\langle N_M \rangle}{C}\right| < \frac{1}{2}\omega_0^2}$$

Condition for resonant trapping during tidal evolution towards synchronicity

$$\left|\frac{\langle N_M\rangle}{C}\right| < \frac{1}{2}\omega_0^2 \quad \Rightarrow \quad \left(\frac{B-A}{C}\right)_{crit} = \frac{5}{2}\frac{k_2}{Q}\left(\frac{R_s}{a}\right)^3 \frac{m_p}{m_s}\frac{1}{|H(p,e)|}$$

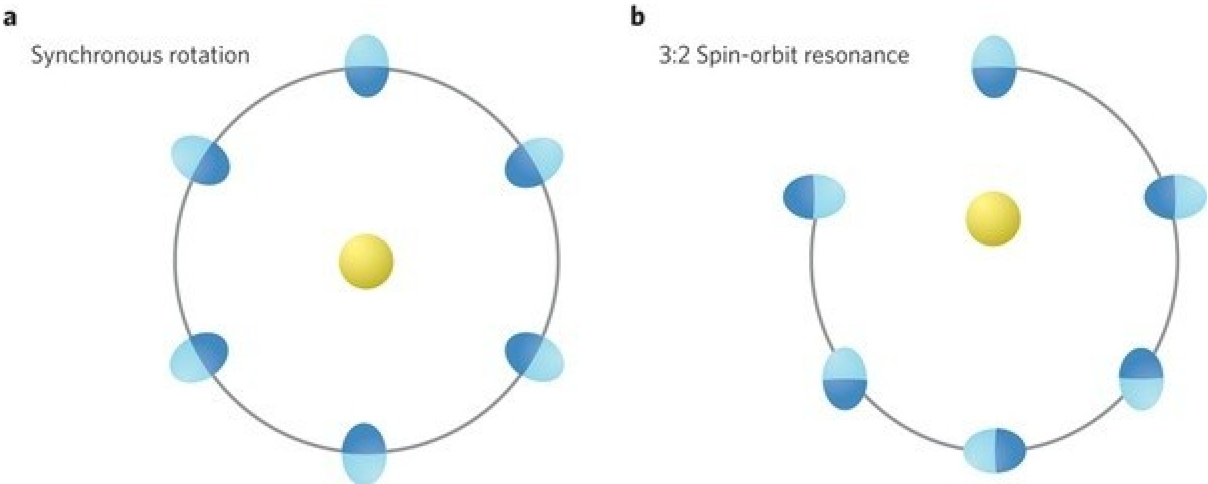
k_2 = Love number tells how much a body is deformed

Q = dissipation function

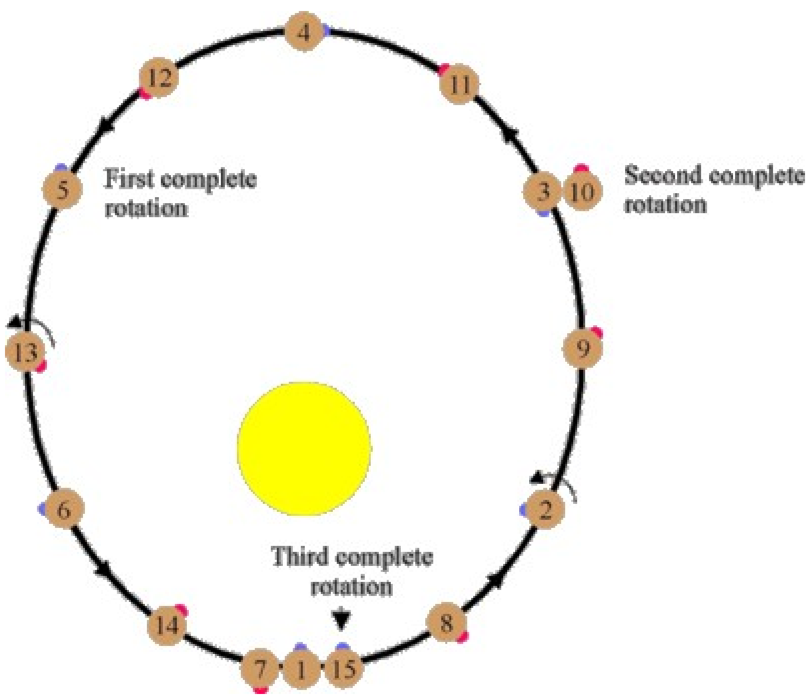
Res p	Mercury $\left(\frac{B-A}{C}\right)_{crit}$	Moon $\left(\frac{B-A}{C}\right)_{crit}$
3	2×10^{-8}	7×10^{-5}
$\frac{5}{2}$	7×10^{-9}	7×10^{-6}
2	3×10^{-9}	8×10^{-7}
$\frac{3}{2}$	2×10^{-9}	1×10^{-7}
1	1×10^{-9}	2×10^{-8}

Present values for Mercury and Moon: $\frac{B-A}{C} \sim 3\times10^{-4}$

If the rotation rate was MUCH larger than the present one, both Mercury and Moon could have been trapped in some other resonance.



MERCURY



$$T_{rot} = 58.65\ d$$

$$T_{orb} = 87.97\ d$$

$$T_{orb} = \frac{3}{2} T_{rot}$$

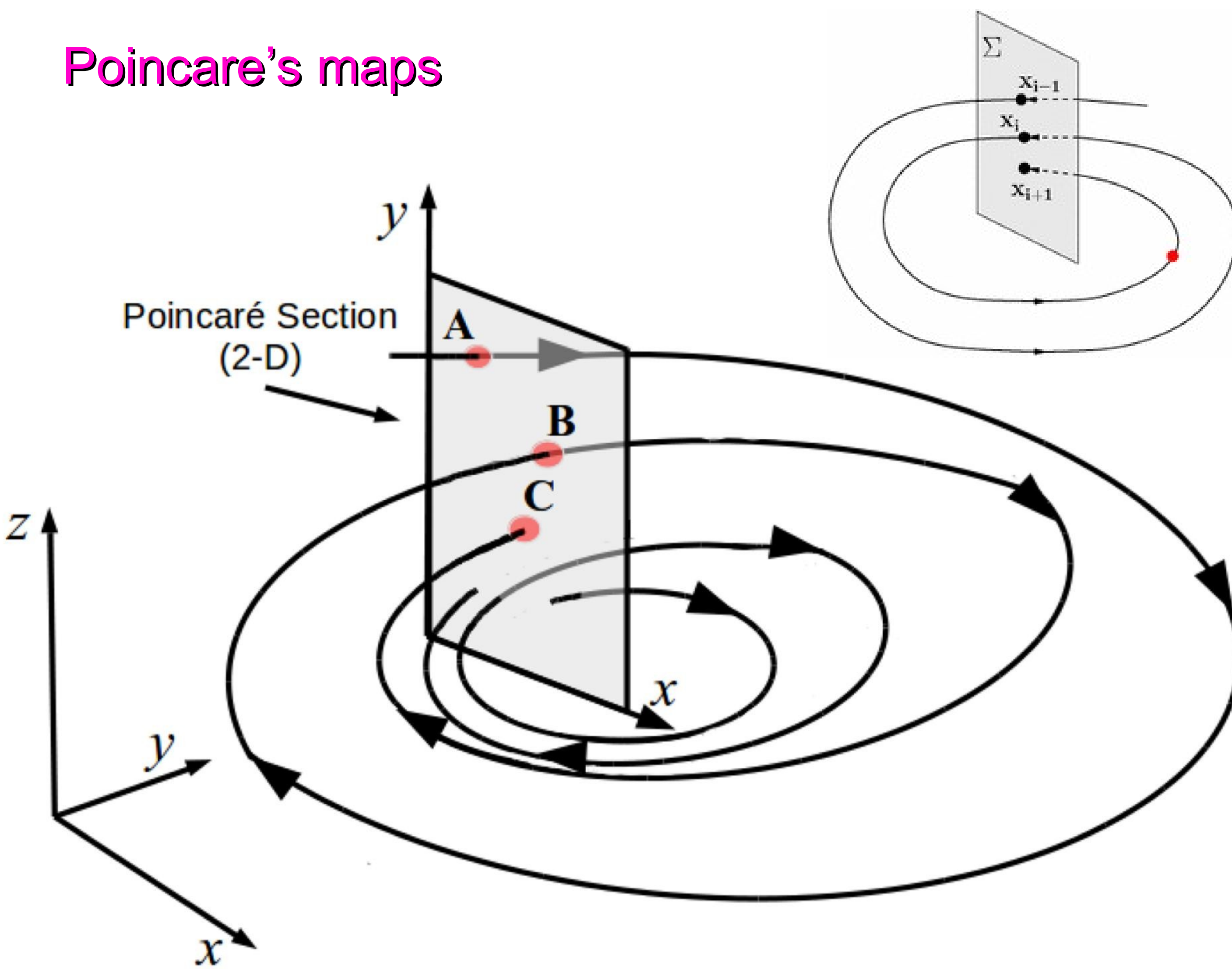
The resonance has halted the tidal evolution!

Stroboscopic effect ....

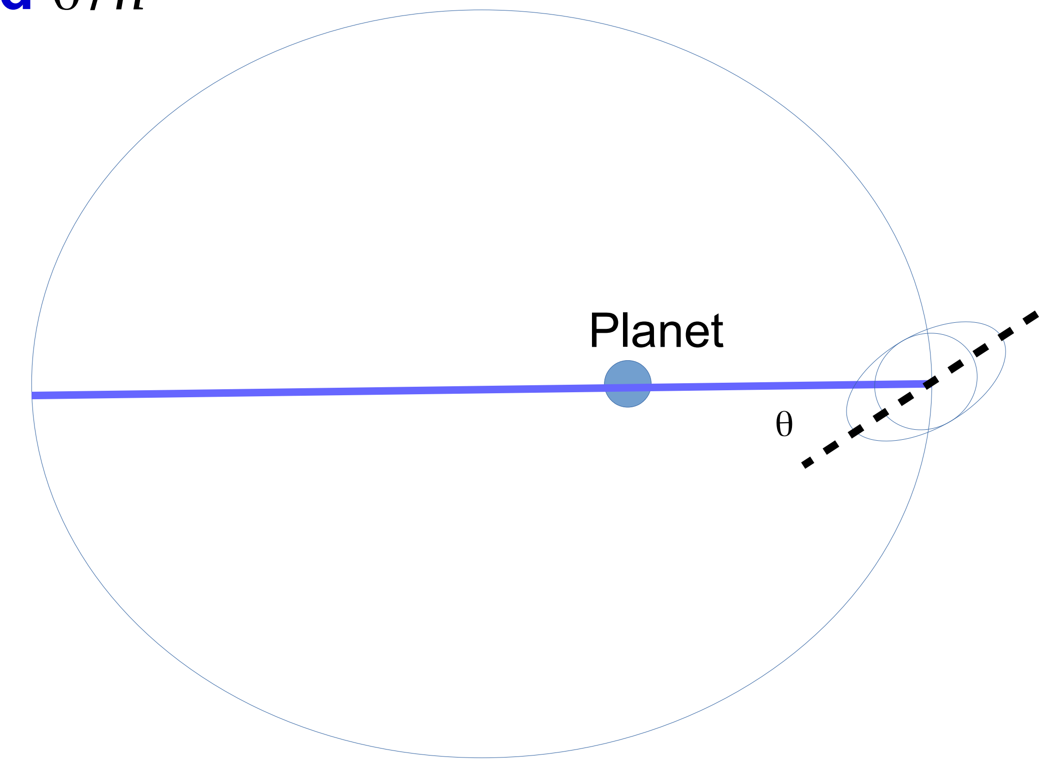
Human eye ~ 24 frames per second for motion, movies ~60 frames per second.

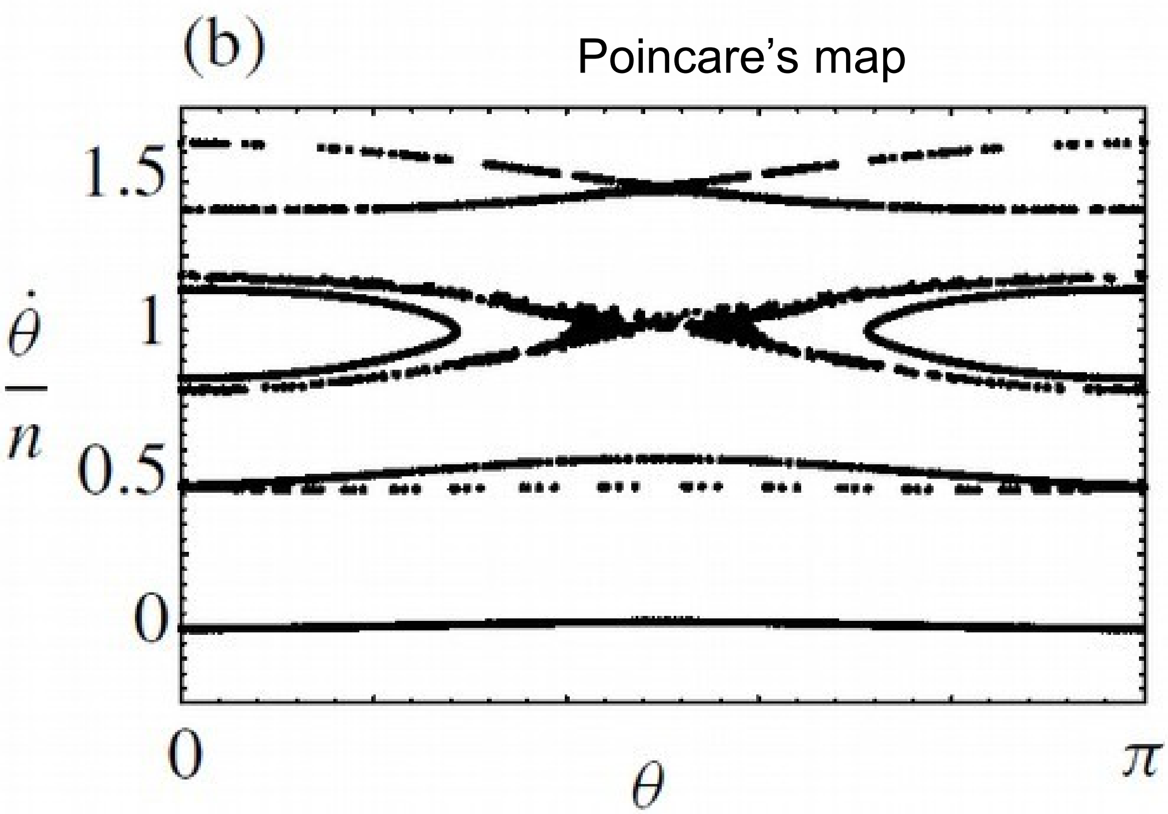
Hamiltonian approach, Poincare's maps, superposition of resonances and chaos….

Poincare's maps



Check the orientation of the satellite any time it passes at the pericenter, then plot θ and $\dot{\theta}/n$





$$\ddot{\theta} = -\frac{3}{2}\left(\frac{B-A}{C}\right) n^2 \left(\frac{a}{r}\right)^3 \sin 2(\theta - f)$$ Equation of motion

The action associated to the angle θ is $\Theta = \dot{\theta}$

$$\dot{\Theta} = -\frac{\partial H}{\partial \theta}$$
$$\dot{\theta} = \frac{\partial H}{\partial \Theta}$$

To obtain the equation of motion from Hamilton equations the Hamiltonian must have the form:

$$H = \frac{\Theta^2}{2} - \frac{3}{4}\left(\frac{B-A}{C}\right) n^2 \left(\frac{a}{r}\right)^3 \cos 2(\theta - f)$$

The angle *f* is periodic in the mean anomaly and also *r*, so the expression can be Fourier expanded in *M*.

$$H = \frac{\Theta^2}{2} - \frac{3}{4}\left(\frac{B-A}{C}\right) n^2 \sum_r K_r(e) \cos(2\theta - rM)$$

The different functions $K_r(e)$ can be computed for the series expansion

From Celletti, Journal of Applied Mathematics and Physics 41, 453, 1990

$$H = \frac{\Theta^2}{2} - \frac{3}{2}\left(\frac{B-A}{C}\right) n^2 [$$

$$\left(-\frac{e}{4} + \frac{e^3}{32}\right)\cos(2\theta - M) + \qquad r=1 \quad 2:1$$

$$\left(\frac{1}{2} - \frac{5}{4}e^2 + \frac{13}{32}e^4\right)\cos(2\theta - 2M) + \qquad r=2 \quad 1:1$$

$$\left(\frac{7}{4}e - \frac{123}{32}e^3\right)\cos(2\theta - 3M) + \qquad r=3 \quad 2:3$$

$$\left(\frac{17}{4}e^2 - \frac{115}{12}e^4\right)\cos(2\theta - 4M) + \qquad r=4 \quad 1:2$$

$$\left(\frac{845}{96}e^3 - \frac{32525}{1536}e^5\right)\cos(2\theta - 5M) + \qquad r=5 \quad 2:5$$

$$\ldots\ldots\ldots\ldots\ldots]$$

p	r
3	6
5/2	5
2	1
3/2	3
..	..

Earth – Moon system

Short tutorial on canonical transformations

Definitions: A coordinate transformation from *(p,q)* to *(w,z)*, where p and w are the momenta, is canonical if

1) it preserves the Hamiltonian form of the equations of motion

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad H(p,q) = k(w,z) \qquad \dot{z}_i = \frac{\partial K}{\partial w_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad \dot{w}_i = -\frac{\partial K}{\partial z_i}$$

2) It preserves the area, in other words the Jacobian of the transformation has determinant equal to 1

$$det \begin{pmatrix} \frac{\partial w}{\partial p} & \frac{\partial w}{\partial q} \\ \frac{\partial z}{\partial p} & \frac{\partial z}{\partial q} \end{pmatrix} = 1$$

3) A generatrix function of mixed variables exists so that, ad example,

$$p = \frac{\partial F(q,w)}{\partial q} \qquad z = \frac{\partial F(q,w)}{\partial w}$$

A simple example is $F = k\,q\,w$ so that $w = p/k$ and $z = kq$.

The harmonic oscillator equations of motion can be solved by using a canonical transformation. We start from its Hamiltonian

$$H = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2}$$

A suited canonical transformation is generated by the function

$$F(q,z)=\frac{m}{2}\omega q^2 cotg\, z$$

The new variables *w,z* are related to *p,q* in the following way

$$p=\frac{\partial F}{\partial q}=m\omega q\, cotg\, z \qquad q=\pm\sqrt{\frac{2w}{m\omega}}\sin z$$

$$w=-\frac{\partial F}{\partial z}=\frac{m\omega q^2}{2\sin^2 z} \qquad p=\pm\sqrt{2m\omega w}\cos z$$

q is computed from the second equation and is inserted in the first to get p.

The new Hamiltonian is obtained by substituting the expressions for *p,q* in the old Hamiltonian

$$K(w,z)=H(p,q)=\omega w\cos^2+\omega w\sin^2 z=\omega w$$

The Hamilton equations in the new coordinates are

$$\dot{z}=\frac{\partial K}{\partial w}=\omega$$

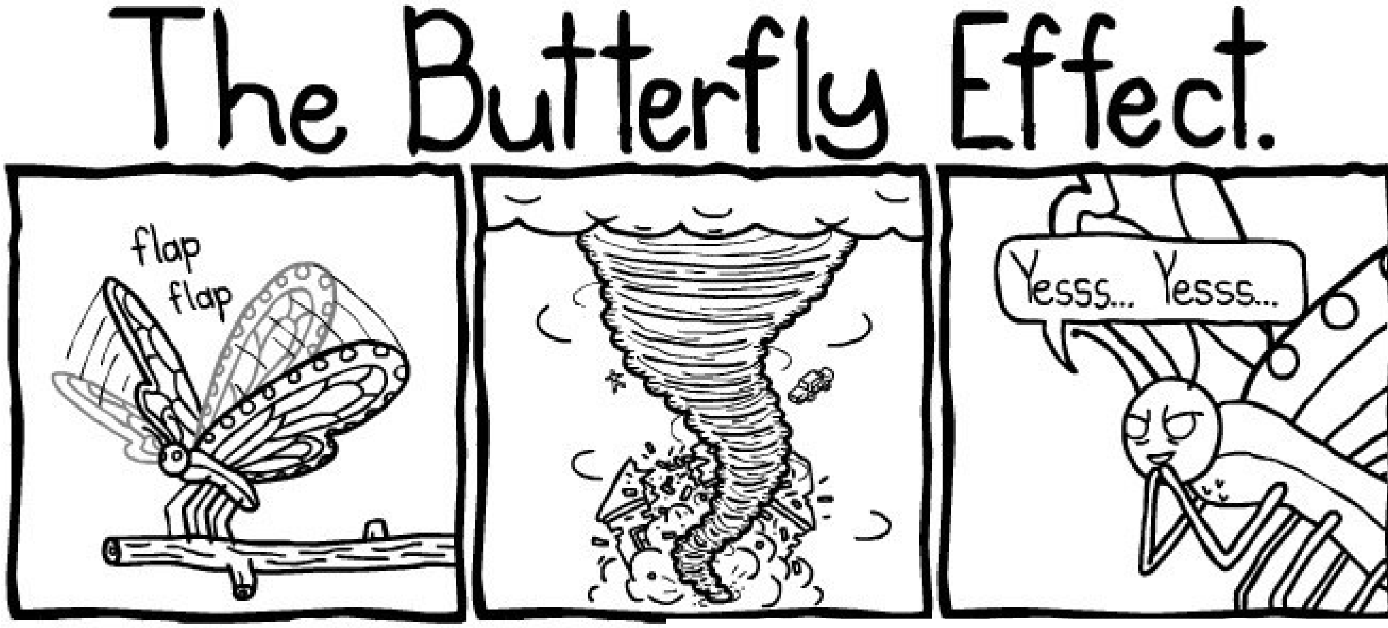
$$\dot{w}=-\frac{\partial K}{\partial z}=0$$

..and the solution is

$$z=\omega t+z_0 \qquad q(t)=\pm\sqrt{\frac{2w_0}{m\omega}}\sin(\omega t+z_0)$$

$$w=w_0 \qquad p(t)=\pm\sqrt{2m\omega w}\cos(\omega t+z_0)$$

Moving back to the old coordinates we have found a solution of the equations of motion. The philosophy is that using a suited canonical transformation we move to coordinates where the Hamiltonian is much simpler (ad example it depends only on momenta) and the Hamilton equation are easy to solve.



Resonance superposition and chaos, Chirikov criterion….

$$H_r = \frac{\Theta^2}{2} - \overline{\omega}_0^2 H(p,e)\cos(2\psi)$$

$$'\overline{\omega}_0^2 = \frac{3}{4}\left(\frac{B-A}{C}\right)n^2 = \frac{1}{4}\omega_0^2$$

Critical angle

$$\psi = \theta - \frac{r}{2}M$$

Canonical transformation (time dependent on M = n t)

$$\Psi = \dot{\theta} - \frac{r}{2}n = \Theta - \frac{r}{2}n$$

$$\psi = \theta - \frac{r}{2}M$$

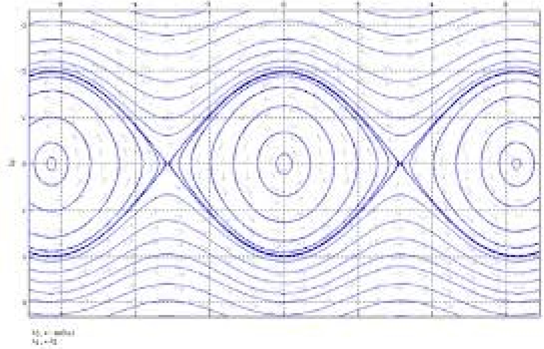
$$det\begin{pmatrix} \frac{\partial\Psi}{\partial\Theta} & \frac{\partial\Psi}{\partial\theta} \\ \frac{\partial\psi}{\partial\Theta} & \frac{\partial\psi}{\partial\theta} \end{pmatrix} = 1$$

Generating function

$$F_2(\theta,\Psi,t) = \Psi\,\theta + \frac{r}{2}n\,\theta - \frac{r}{2}M\,\Psi - \frac{r^2 n M}{8} \qquad M = nt$$

$$\Theta = \frac{\partial F_2}{\partial\theta} = \Psi + \frac{r}{2}n \qquad \psi = \frac{\partial F_2}{\partial\Psi} = \theta - \frac{r}{2}M$$

Resonant Hamiltonian

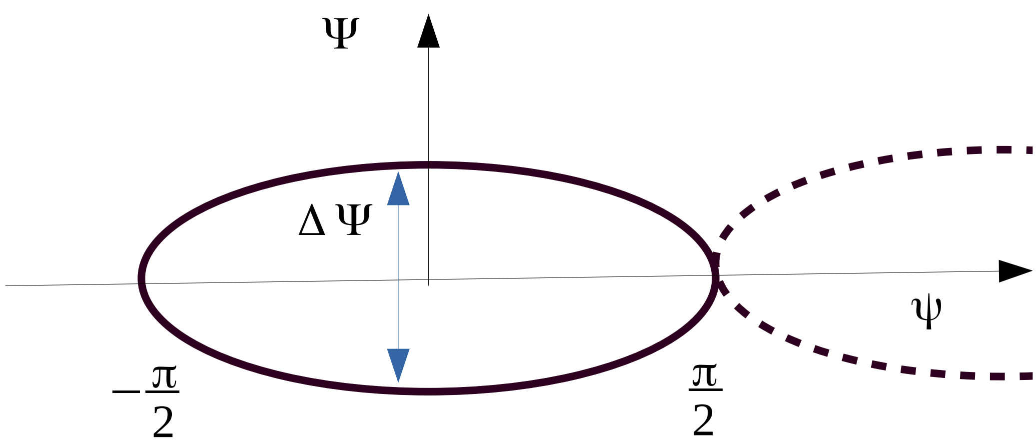


$$K_R = \frac{\Psi^2}{2} - \overline{\omega}_0^2 H(p,e)\cos(2\psi)$$

K_R is the energy constant so in the phase space the trajectories are :

$$\Psi(\psi) = \pm\sqrt{2K_R + 2\overline{\omega}_0^2 H(p,e)\cos(2\psi)}$$

The separatrix has at $\psi = \pm\frac{\pi}{2}$ $\Psi = 0$



...for the resonant condition $\Psi = \dot{\theta} - \frac{r}{2}n = 0$

From the above equation and thanks to the previous condition it is possible to derive the separatrix equation:

$$2K_R - 2\overline{\omega}_0^2 H(p,e) = 0 \quad \Rightarrow \quad K_R = \overline{\omega}_0^2 H(p,e)$$

The separatrix is:

$$\Psi(\psi)=\pm\sqrt{2\,\overline{\omega}_0^2 H(p,e)(1+\cos(2\psi))}$$

The width of the resonance can then be computed at $\psi=0$

And it is given by

$$\frac{\Delta\Psi}{2}=2\,\overline{\omega}_0\sqrt{H(p,e)}=\omega_0\sqrt{H(p,e)}$$

The condition on ω_0 and on the eccentricity e for

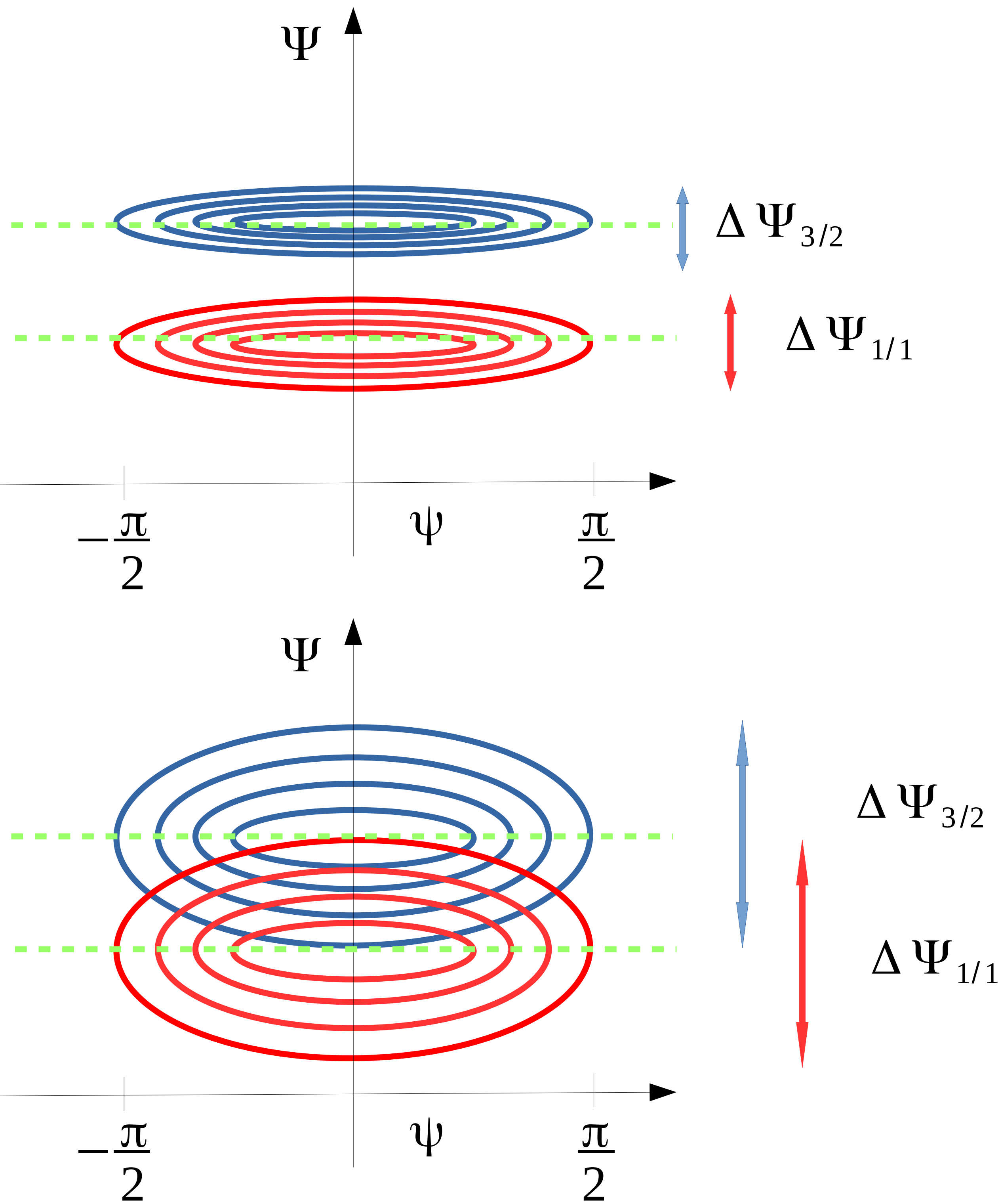
the resonance superposition, ad exampe for the 3/2
and 1/1 resonances, is:

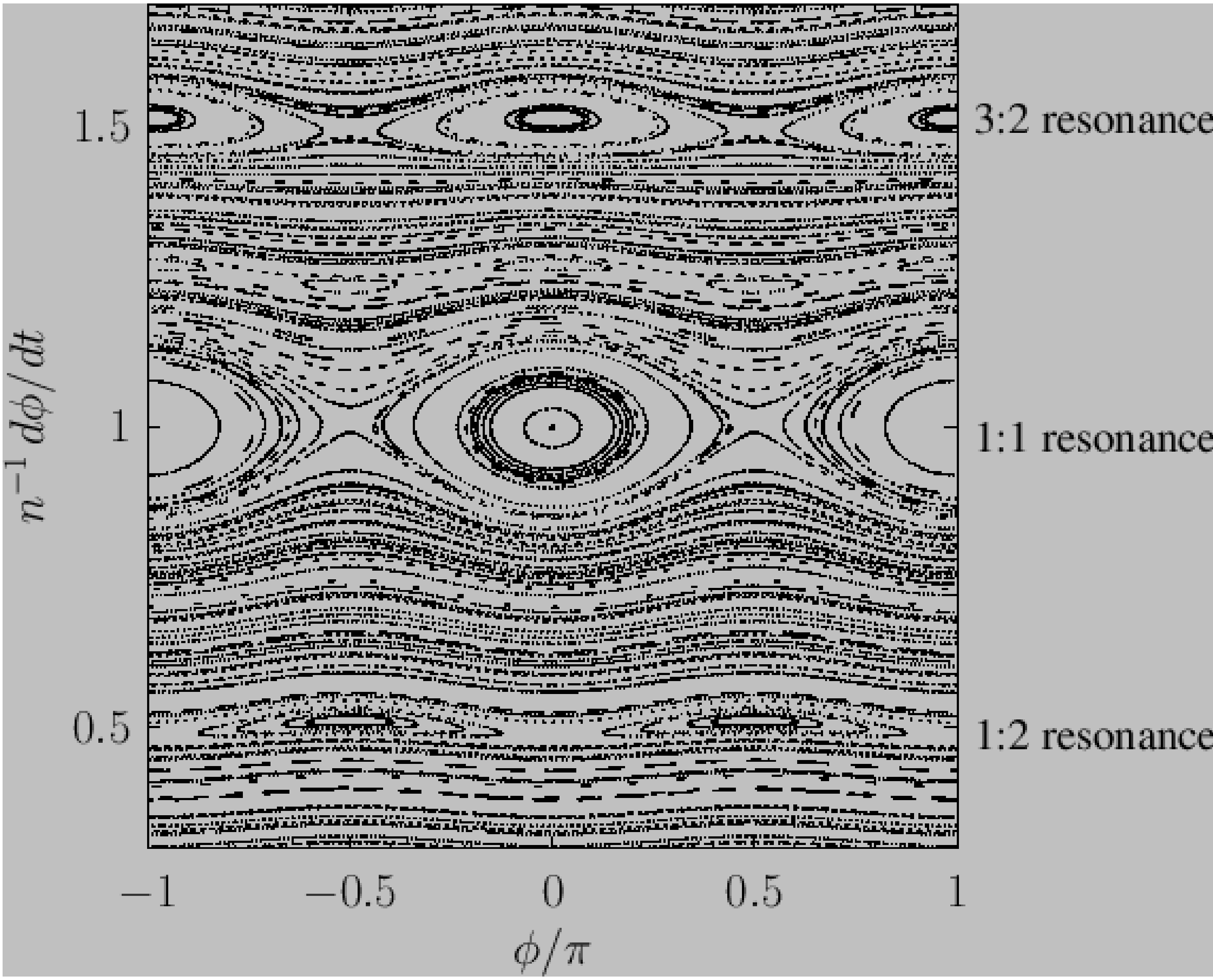
$$\omega_0\sqrt{H(1,e)}+\omega_0\sqrt{H(\frac{3}{2,}e)}=\left(\frac{3}{2}-1\right)n=\frac{1}{2}n$$

$$H_r=\frac{\Theta^2}{2}-\overline{\omega}_0^2 H(1,e)\cos(2\theta-2M)-\overline{\omega}_0^2 H(3/2,e)\cos(2\theta-3M)$$

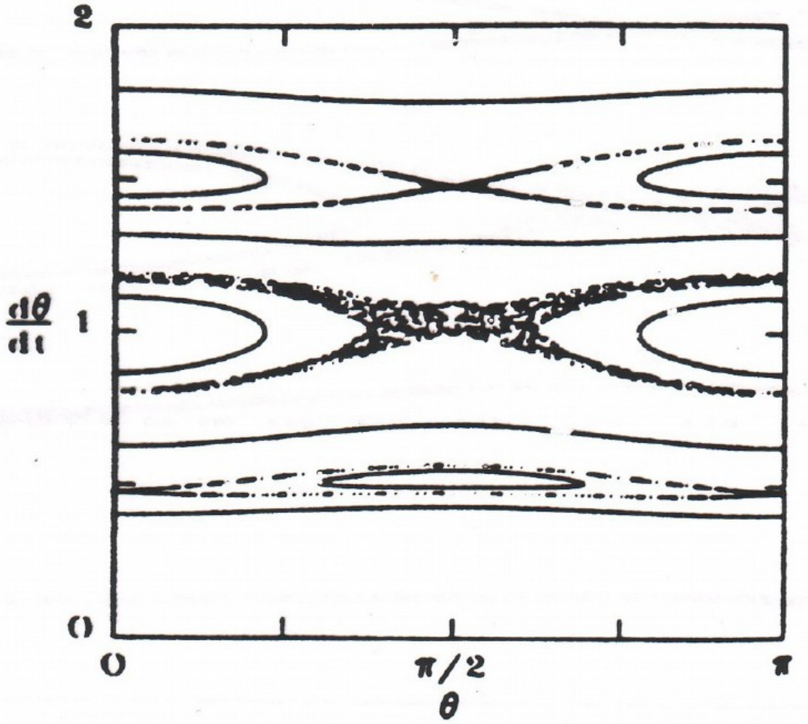
There are two perturbing terms in the Hamiltonian and
their interaction causes chaotic evolution. The system
‘does not know’ which resonance to be in….

For larger values of H(p,e) and ω_0 the width of the resonance increases and superposition can occur generating chaotic evolution.





Numerical maps obtained solving the equations of motion.



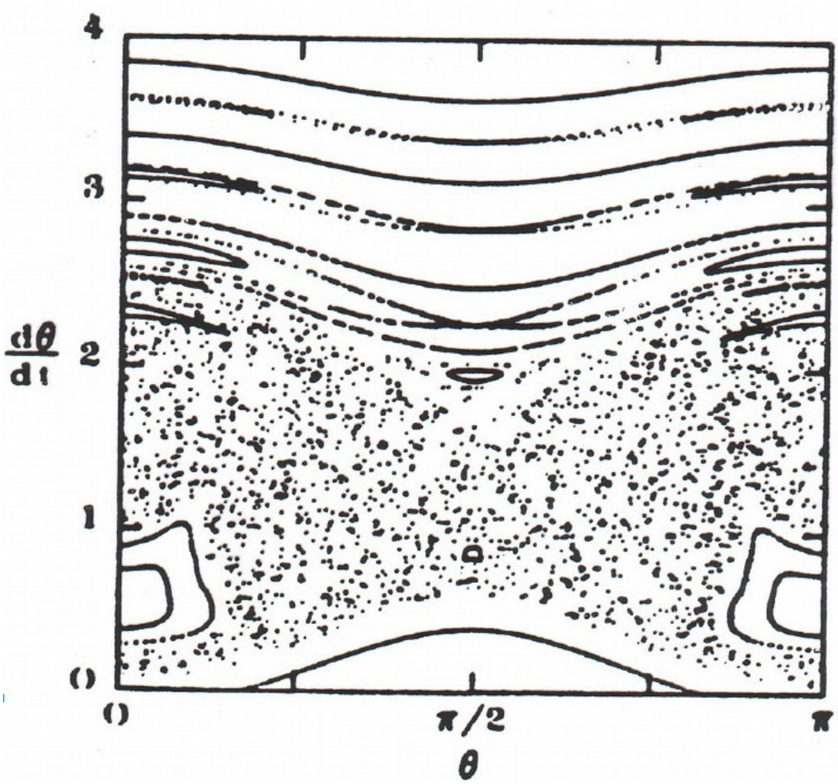
3/2

1/1

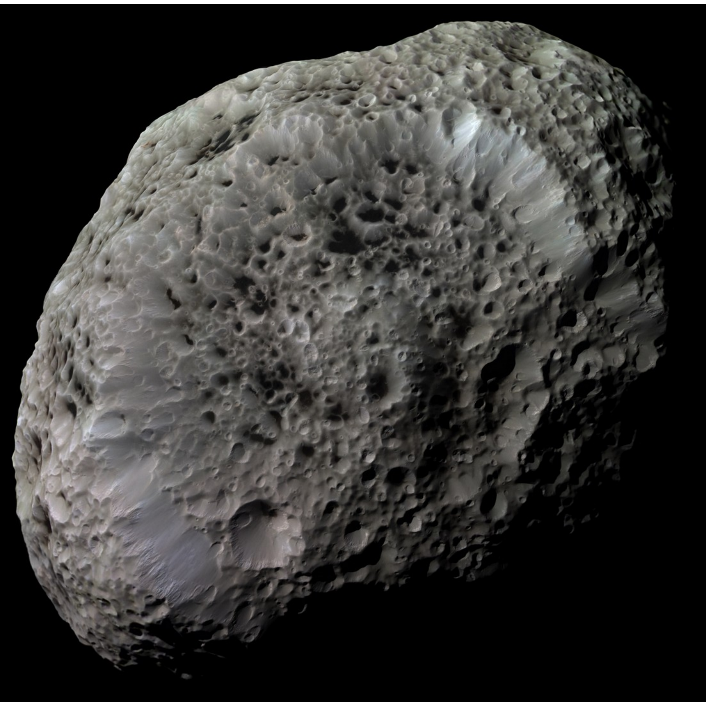
1/2

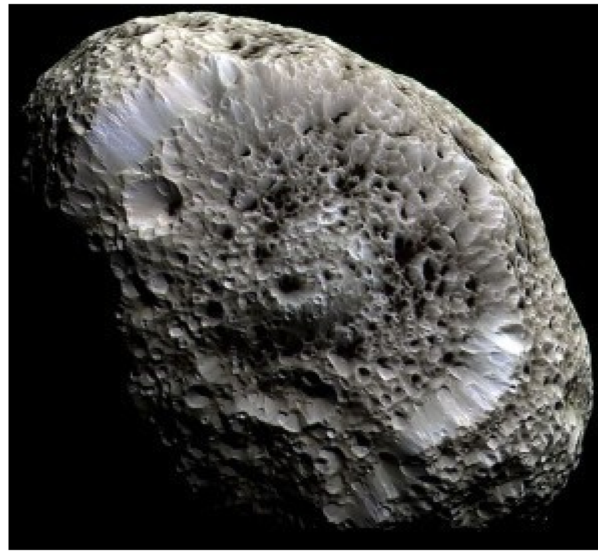
Phase space for small values of eccentricity e. The resonances are well separated

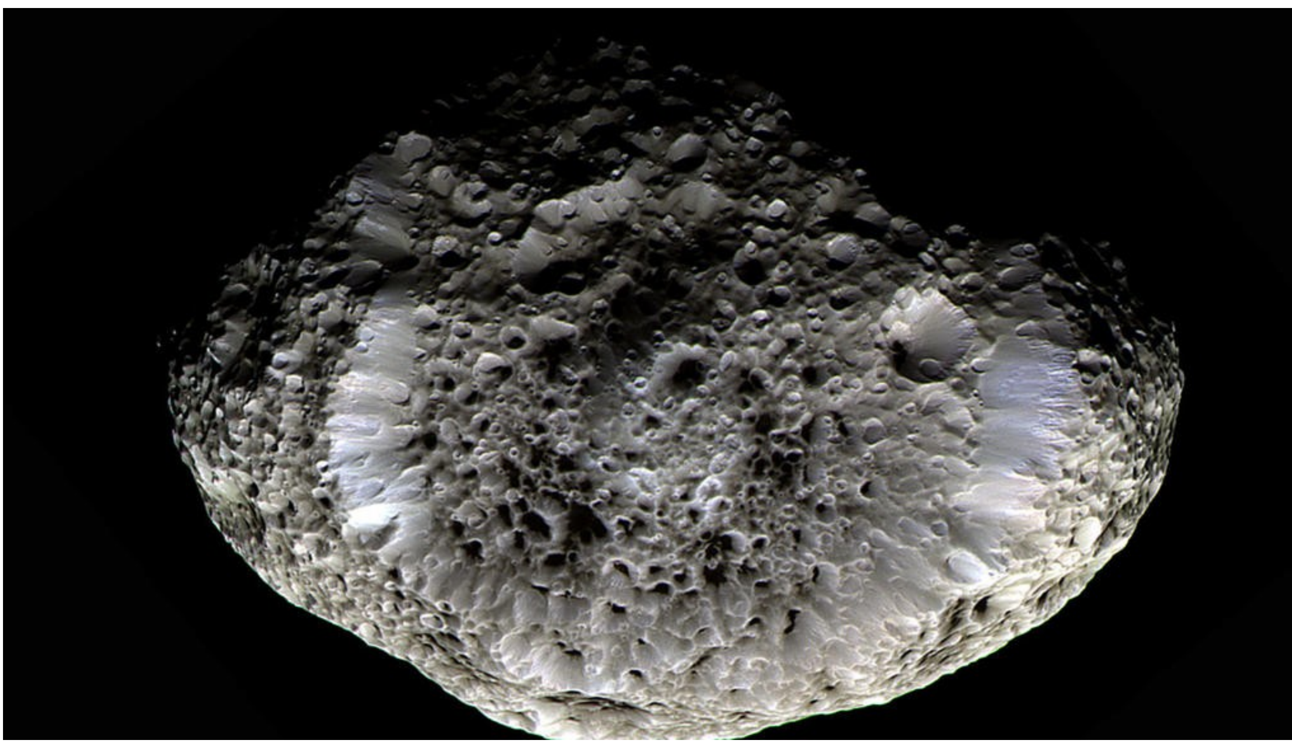
For higher values of eccentricity e, the resonances merge and a large chaotic zone is generated.



Among present satellites, Hyperion is chaotic, Miranda and Mimas (syncronus rotation) may have been chaotic in the past.







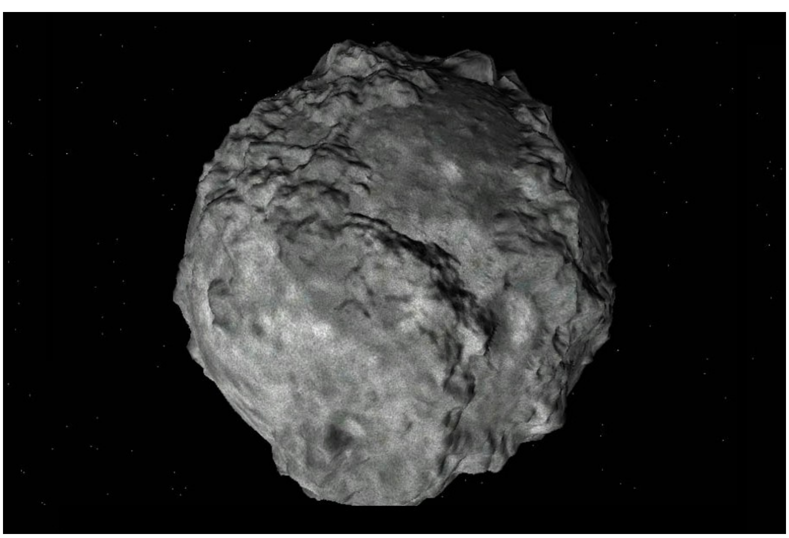
Hyperion, Saturn satellite, very irregular and large
(B-A)/C   360.2 km × 266 km × 205.4 km

$\omega_0^{RO} = 0.31$ **$\omega_0 \sim 0.8$**   **Chaotic!!**

Neptune satellite system

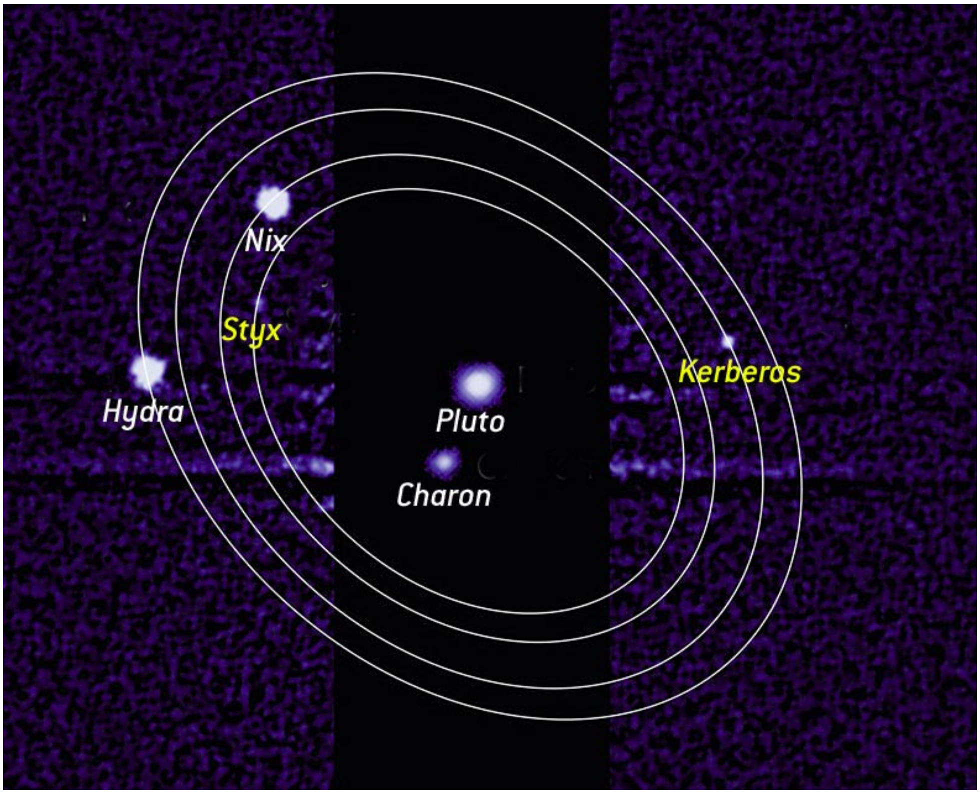
Orbital parameters

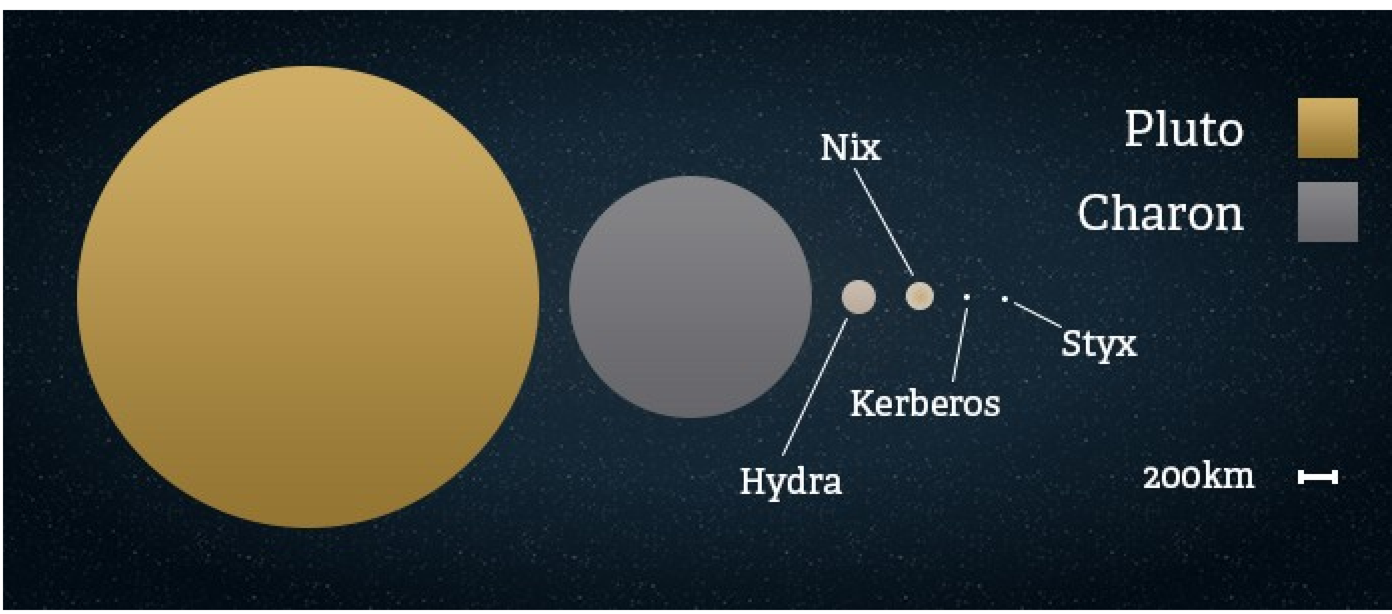
	Semi-major axis (10^3km)	Semi-major axis (Neptunian Radii)	Orbital Period* (days)	Rotation Period (days)	Inclinatio (degrees)
Naiad (NIII)	48.227	1.948	0.294396		4.74
Thalassa (NIV)	50.075	2.022	0.311485		0.21
Despina (NV)	52.526	2.121	0.334655		0.07
Galatea (NVI)	61.953	2.502	0.428745		0.05
Larissa (NVII)	73.548	2.970	0.554654		0.20
S/2004 N1	105.300	4.252	0.950		0.00
Proteus (NVIII)	117.647	4.751	1.122315		0.04
Triton (NI)	354.76	14.328	5.876854R	S	157.345
Nereid (NII)	5,513.4	222.67	360.13619		7.23
Halimede (NIX, S/2002 N1)	15730.	635.2	1879.7R		134.1
Sao (NXI, S/2002 N2	22420.	905.3	2914.1		48.5
Laomedeia (NXII, S/2002 N3)	23570.	951.8	3167.9		34.7
Psamathe (NX, S/2003 N1)	46700.	1885.8	9115.9R		137.4
Neso (NXIII, S/2002 N4)	48390.	1954.0	9374.0R		132.6



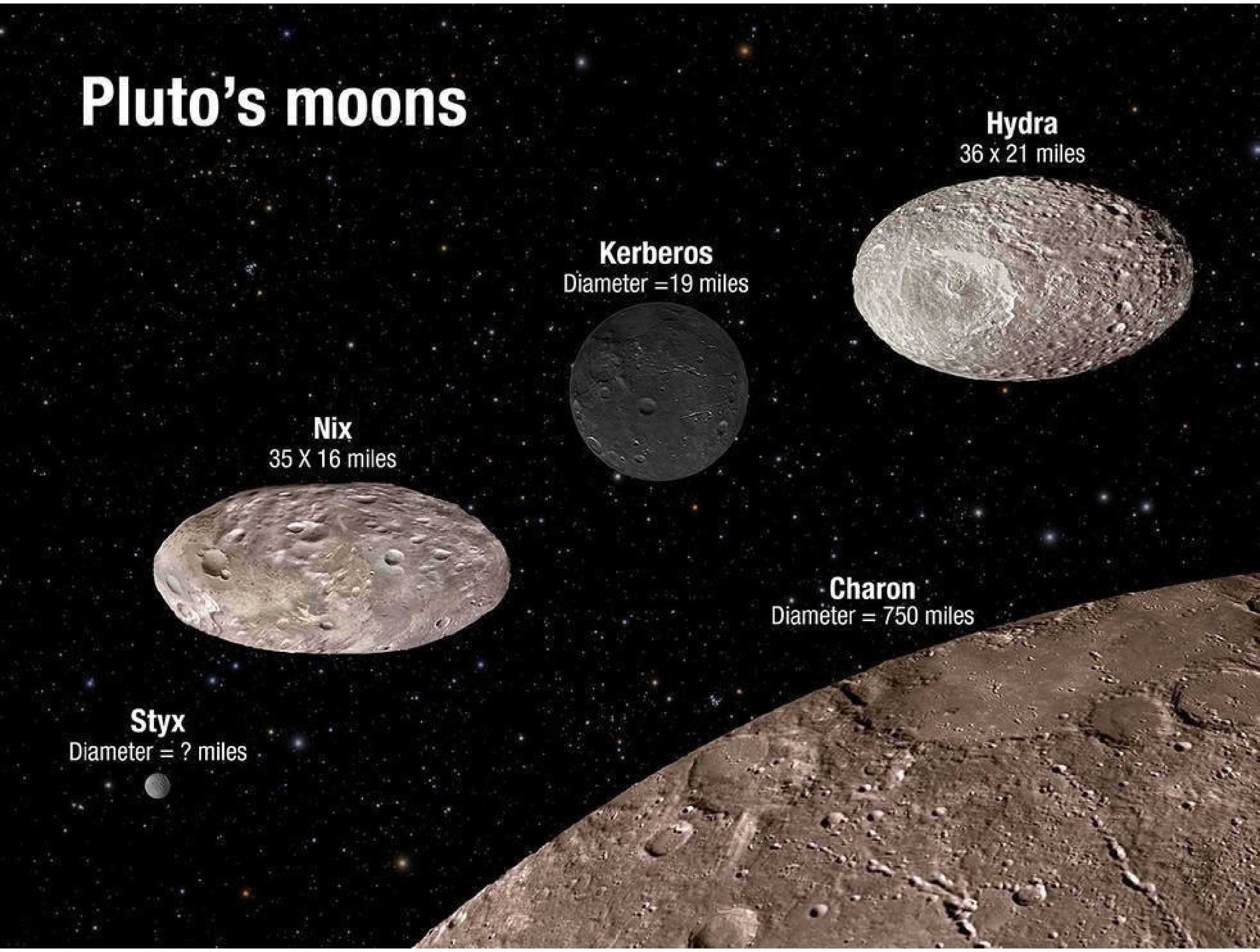
Nereid, satellite of Neptune (400 km of diameter) has an eccentricity e = 0.751, it could be chaotic!

Pluto moons

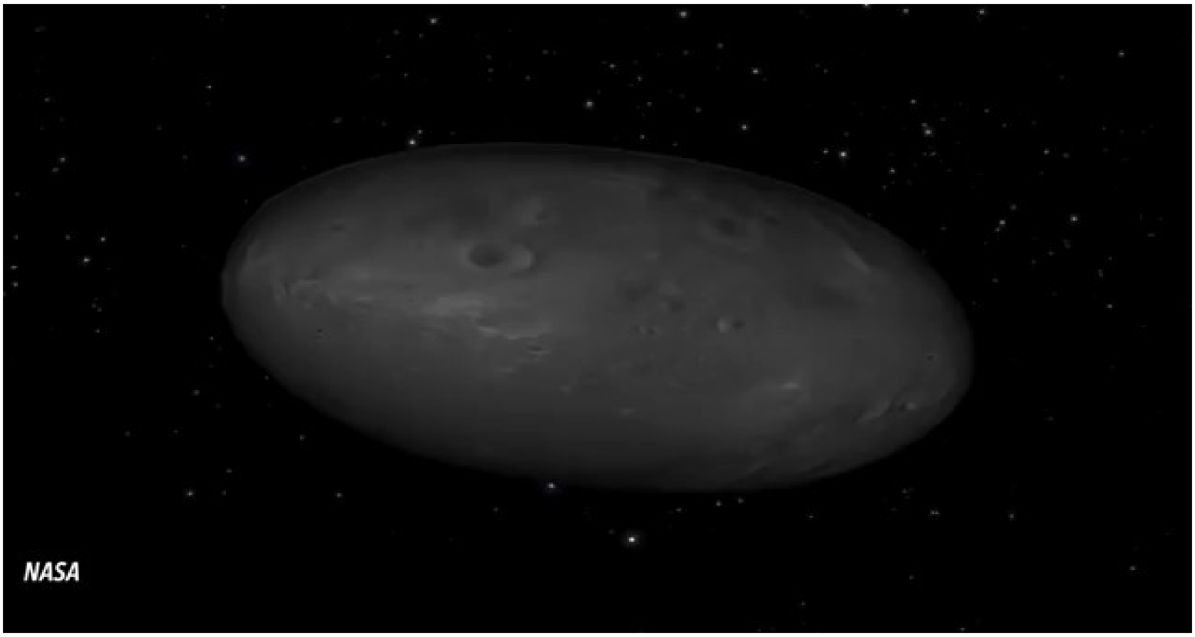




Styx, Hydra e Nix are in a 3-body resonance like Io, Europa and Ganymed.

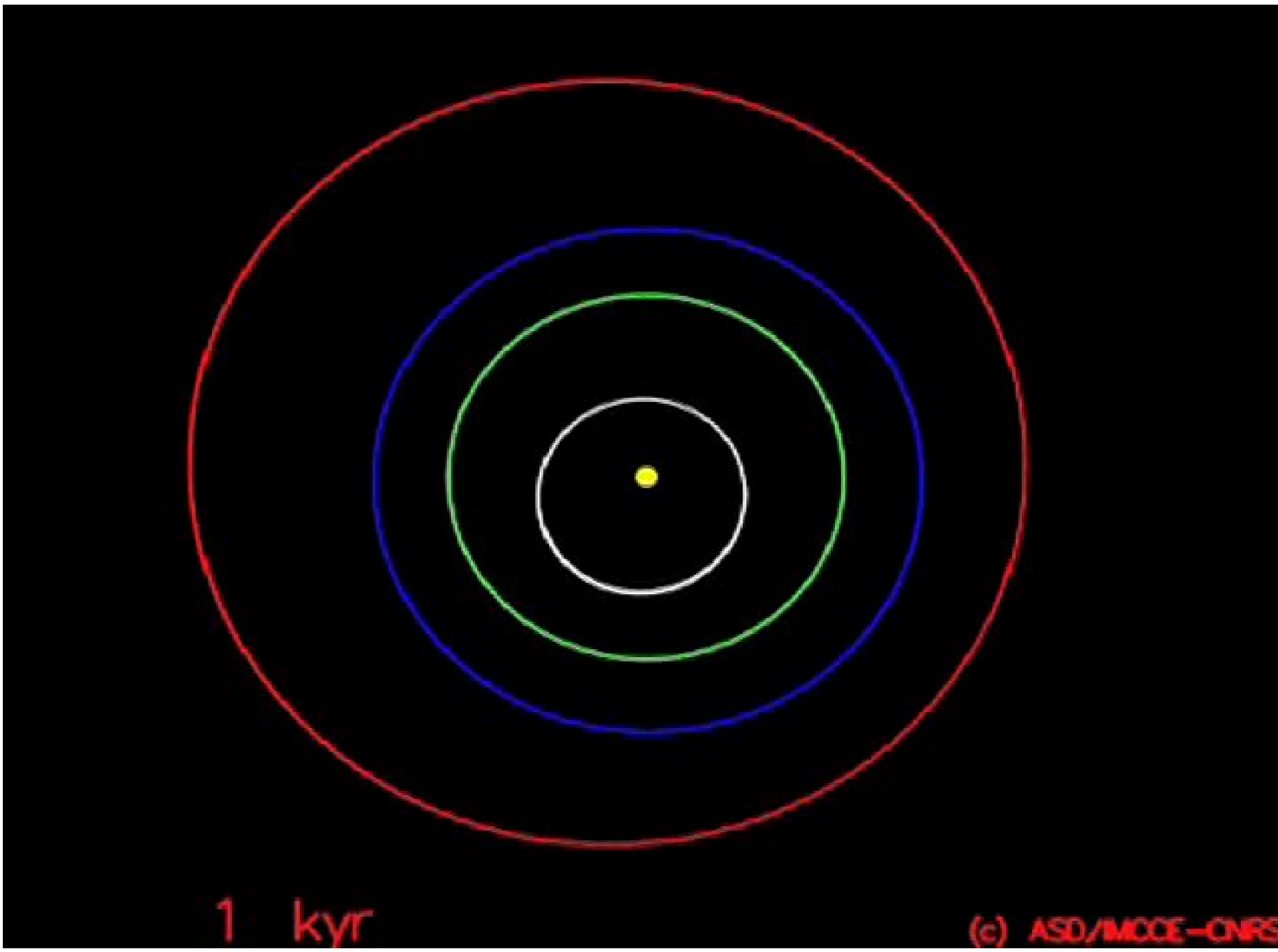

Pluto's moons
Hydra
36 x 21 miles
Kerberos
Diameter =19 miles
Nix
35 X 16 miles
Charon
Diameter = 750 miles
Styx
Diameter = ? miles

Chaotic tumbling: also the **obliquity** of Nix evolves chaotically.



Behaviour deduced from lightcurves.
Probably also Styx, Kerberos and Hydra are chaotic.

Also the Solar System is chaotic. On the long term (Gyrs) the orbit of Mercury may destabilize and drive into chaos the inner solar system. Possible collisions between the terrestrial planets.



Obliquity of Jupiter (~3°) and Saturn (~27°) due to resonance crossing with the fundamental frequencies of the solar system g7 and g8 during planet migration.

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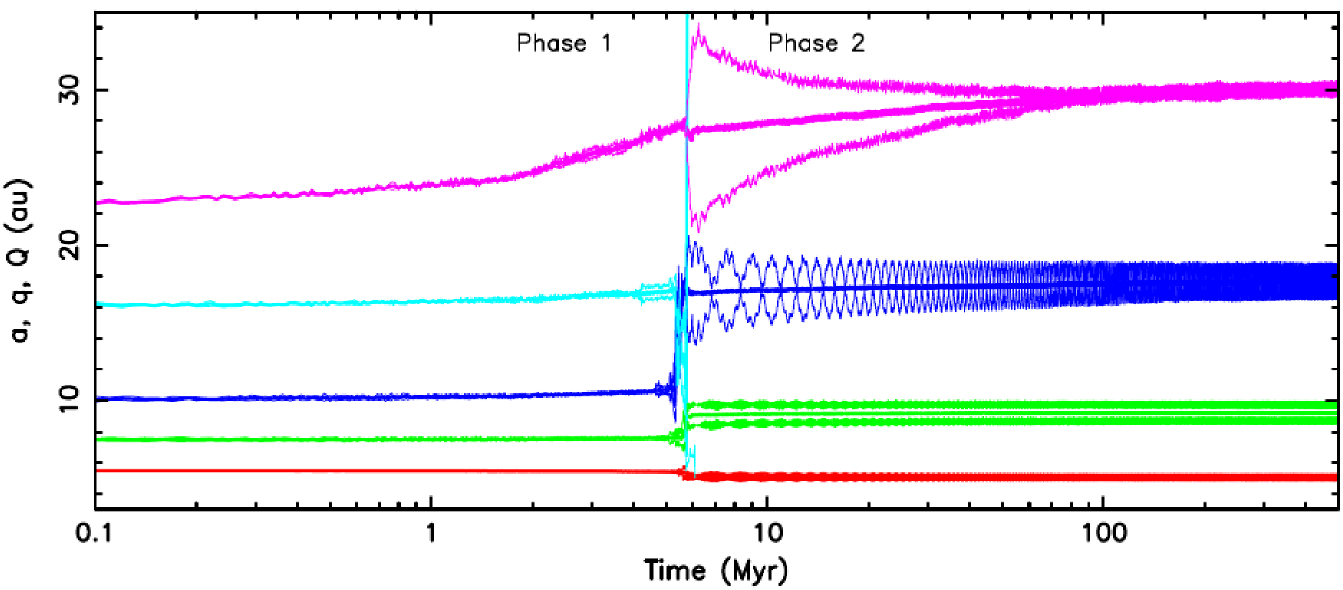
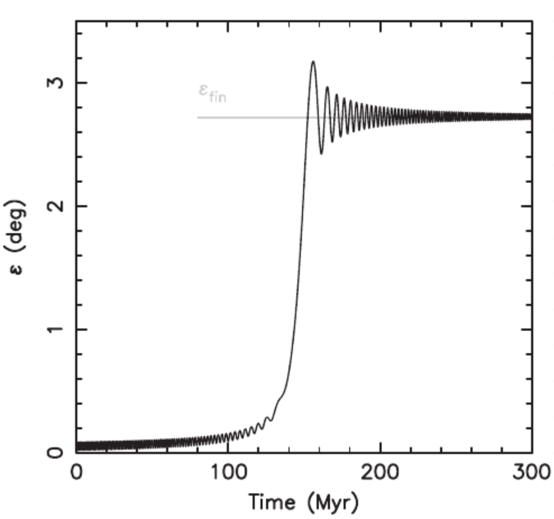


Giove



Saturno