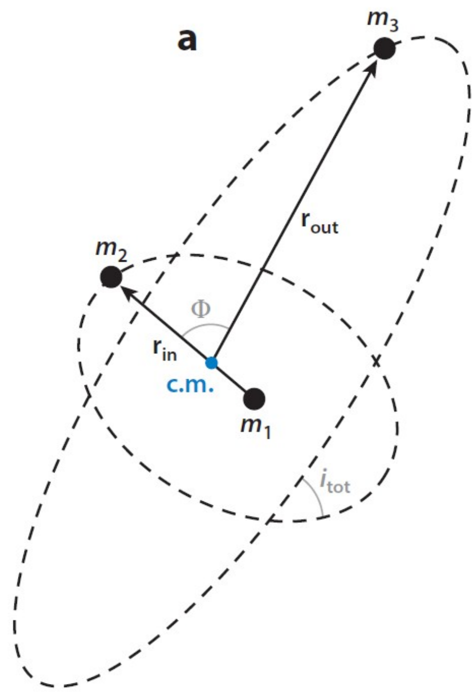
Kozai-Lidov



Full 3-body problem with the outer body far out respect to the inner pair so that

$$\alpha = \frac{a_1}{a_2}$$

is a **small parameter.**

The full Hamiltonian (including the Keplerian terms):

$$F = \frac{G m_1 m_2}{2 a_1} + \frac{G m_3 (m_1 + m_2)}{2 a_2} + \frac{G}{a_2} \sum_{n=2}^{\infty} \left(\frac{a_1}{a_2}\right)^n M_n \left(\frac{r_1}{a_1}\right)^n \left(\frac{a_2}{r_2}\right)^{(n+1)} P_n(\cos(\phi))$$

Where the constant mass term is

$$M_n = m_1 m_2 m_3 \frac{m_1^{n-1} - (-m_2)^{n-1}}{(m_1 + m_2)^n}$$

a_1 is the semimajor axis of the orbit of 2 vs. 1, while a_2 is the semimajor axis of body 3 vs. CM of 1 and 2.

Naoz, ARAA 2016

In the simplified problem where the second body is treated as a **massless test particle** in the presence of an external massive perturber (star or planet) on a **fixed orbit**, a set of canonical coordinates (assuming that the outer body is on a fixed orbit) are given by:

$$L=\frac{m_1 m_2}{m_1+m_2}\sqrt{\gamma^2(m_1+m_2)a} \qquad M$$

$$G=L\sqrt{(1-e^2)} \qquad \omega$$

$$H=G\cos(i)=L\sqrt{(1-e^2)}\cos(i) \qquad \Omega$$

The perturbative **Hamiltonian** (without the Keplerian terms) is:

$$F = F_{qu} + \epsilon F_{oc} \qquad \epsilon = \frac{(a/a_2)e_2}{1-e_2^2} \qquad a = a_1 \quad e = e_1$$

$$F_{qu} = -\frac{e^2}{2} + \theta^2 + \frac{3}{2}e^2\theta^2 + \frac{5}{2}e^2(1-\theta^2)\cos(2\omega)$$

where $\theta = \cos i$ With **i** the inclination respect to the outer body orbital plane.

$$F_{oc} = \frac{5}{16}\left(e + \frac{3}{4}e^3\right)\{(1+11\cdot\theta - 5\cdot\theta^2 - 15\theta^3)\cos(\omega+\Omega)$$
$$+(1-11\cdot\theta-5\cdot\theta^2+15\cdot\theta^3)\cos(\omega-\Omega)\}$$
$$-\frac{175}{64}e^3\{(1-\theta-\theta^2+\theta^3)\cos(3\omega-\Omega)$$
$$+(1+\theta-\theta^2-\theta^3)\cos(3\omega+\Omega)\}$$

If e_2 is small , the term ε is also small and the quadrupole approximation works well and the Hamilton equation leads to

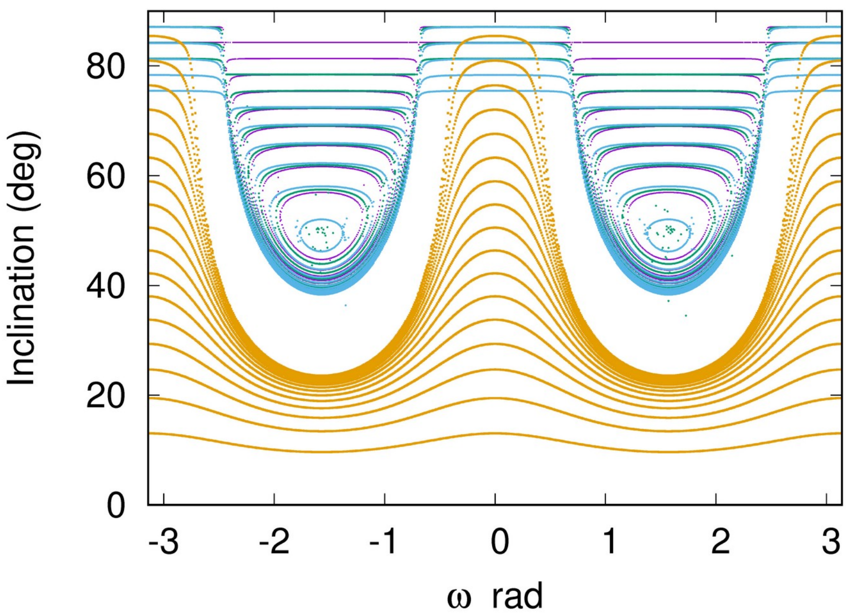
$$\frac{dJ_z}{dt} = \frac{\partial F}{\partial \Omega} = 0$$

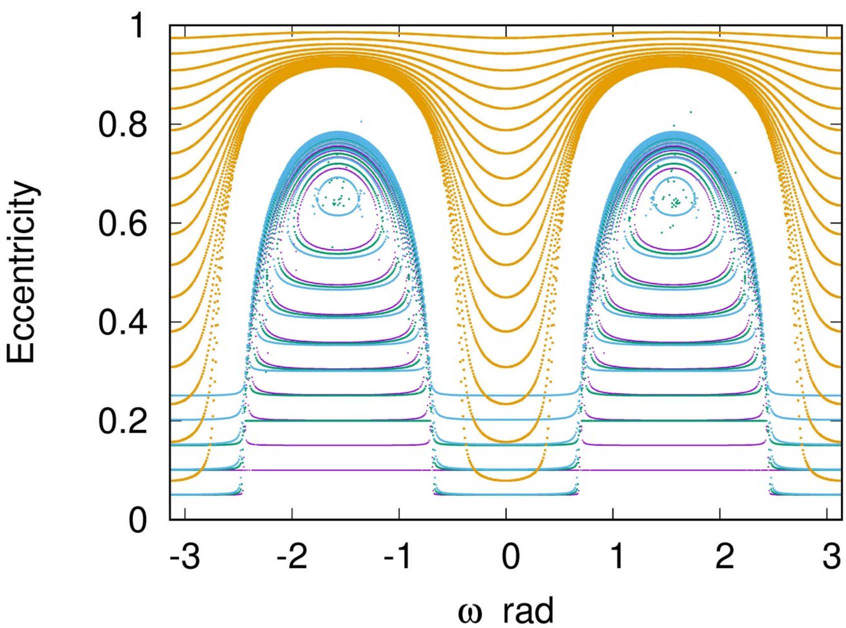
Since F_{qu} does not depend on Ω, J_z is a constant of motion (together with F).

$J_z = \sqrt{(1-e^2)}\,\theta$

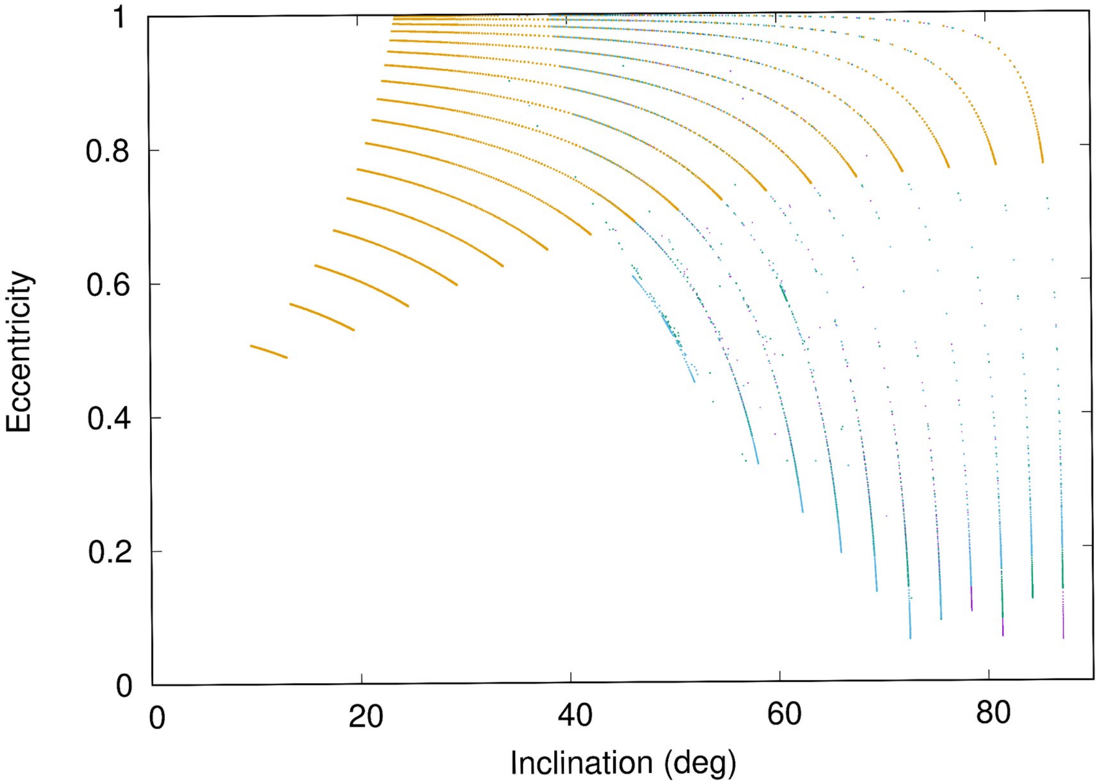
Assuming that J_z is rescaled with

$$m G M_s a = 1$$



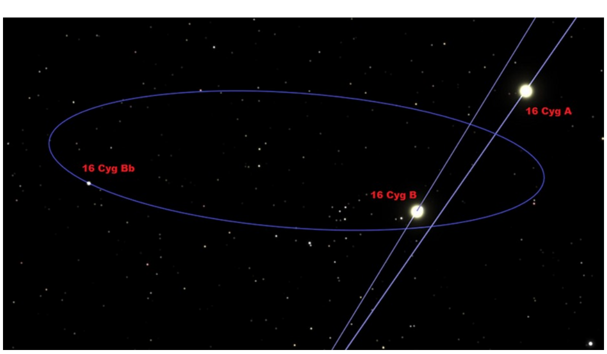


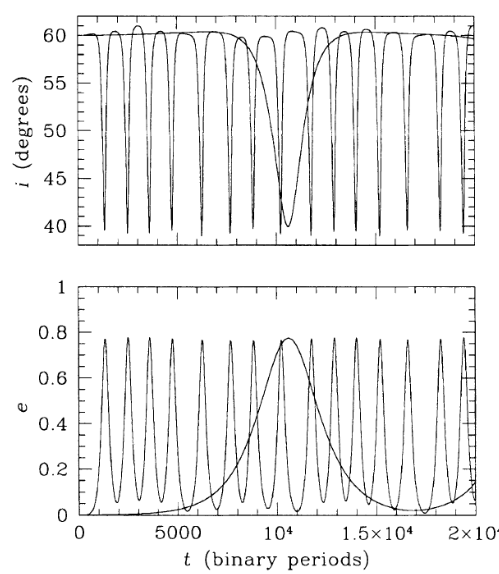
Level curves for given values of J_z and F.



For large values of inclination, small values of eccentricity and viceversa. An external perturber can force eccentricity in exoplanets (16 Cyg) or favor circularization of the orbit by pushing the pericenter of the planet very close to the star where the tidal interaction with the star can act.

Holman et al. 1997





When the eccentricity of the perturber is no longer negligible, than

$$\epsilon = \frac{(a/a_2)\,e_2}{1-e_2^2} > 0$$

And the octupole term comes into play!

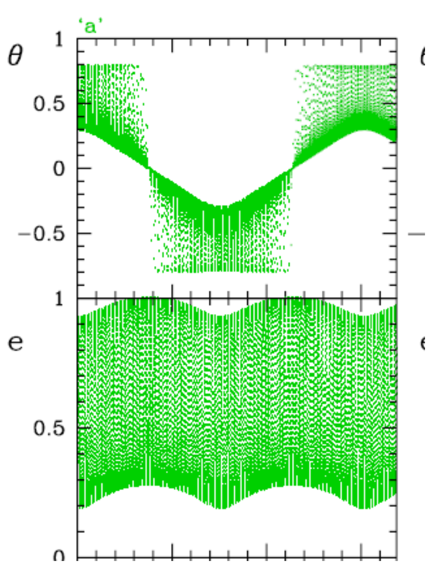
Spin flips: the orbit may become retrograde and viceversa

Onset of chaotic behaviour for increasing α

i=0°

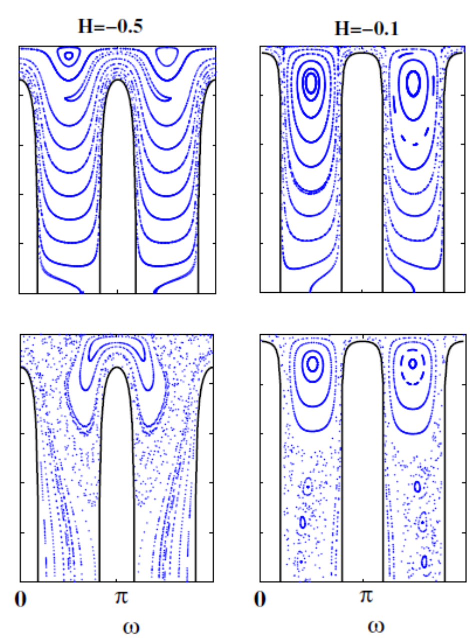
i=90°

i=180°

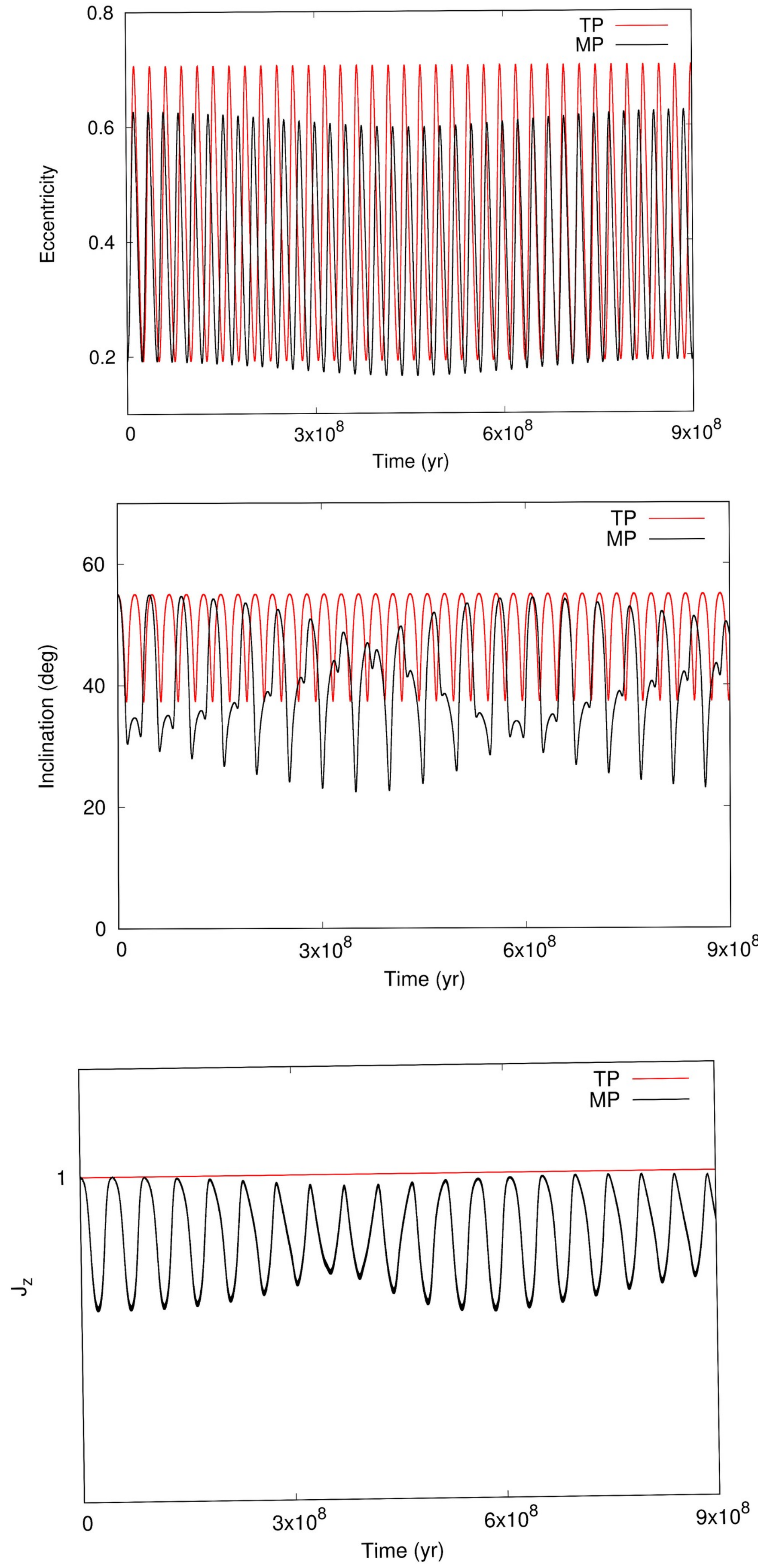


J

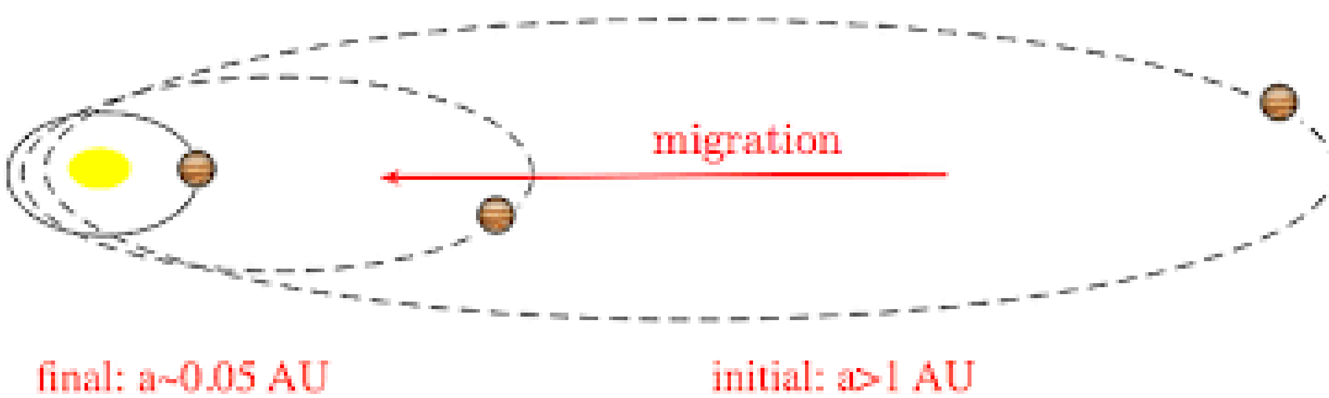
J



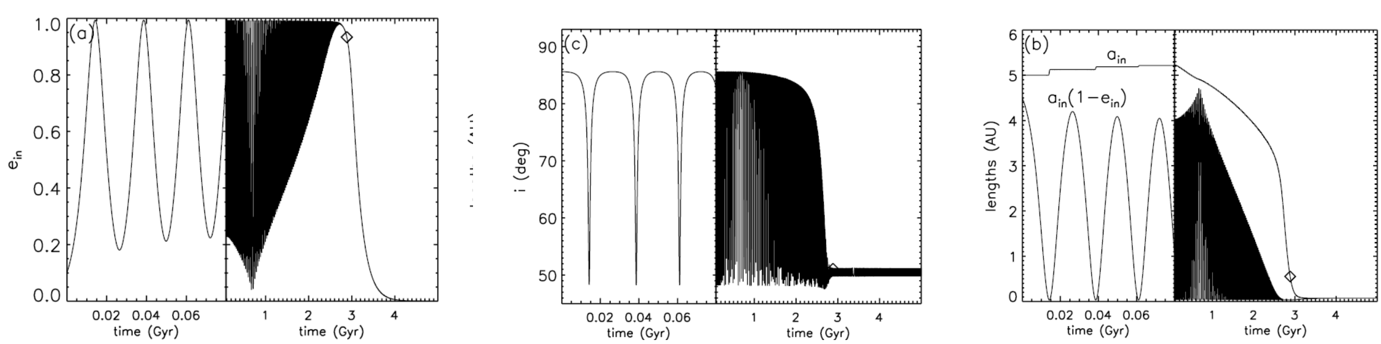
If we relax the assumption of 0 mass for the inner body, then the dynamical evolution is more complex. Ad example, two planets of similar mass have a different behaviour from that described by the quadrupole term in the massless approximation. In particular J_z is not anymore a constant of motion. The constant of motion is now the full angular momentum of the system and the J_z of the outer planet changes accordingly to that of the inner planet to keep the total J constant. In the figures comparison betwen the test particle case (TP) and the massive one (MP) with the two planets having the same mass (= M Jupiter).



Kozai migration of exoplanets: Kozai cycles + tide



It explains also misalignment with respect to the star rotating axis **(Fabryky and Tremaine (2007)).**



Planet initially at a=5 au, e=0.1 while the binary is inclined by 85° and has an eccentricity of 0.5.

Is it possible to switch off the Lidov-Kozai perturbations?

- **General Relativity**
- **Tides**

$$t_{rel} \sim 2\pi \frac{a_1^{5/2} c^2 (1-e_1^2)}{3 G^3 (m_s + m_1)^{3/2}}$$

$$t_{quad} \sim \frac{2\pi a_2^3 (1-e_2^2)^{3/2} \sqrt{(m_1+m_2)}}{a_1^{3/2} m_3 G}$$

Derived from the computation of dω/dt. The relativistic term is derived from the PN approximation.The ratio between the two timescales is:

$$\frac{t_{rel}}{t_{quad}} = \frac{a_1^4}{3 a_2^3} \frac{c^2 (1-e^2) m_2}{(1-e_2^2)^{3/2} (m_1+m_2)^2 G^2}$$

If $t_{rel} < t_{quad}$ then the relativistic term dominates and the Kozai Lidov oscillations are damped.

The effects of tides is complex:

- **It causes precession of perihelia**
- **It damps the eccentricity**
- **There are tides of the star on the planet and vicevers**
- **Dynamic and static tides.**

Comparing the precession due to Lidov-Kozai to that due to tides and also including the eccentricity damping appears to be difficult from an analytical point of view.