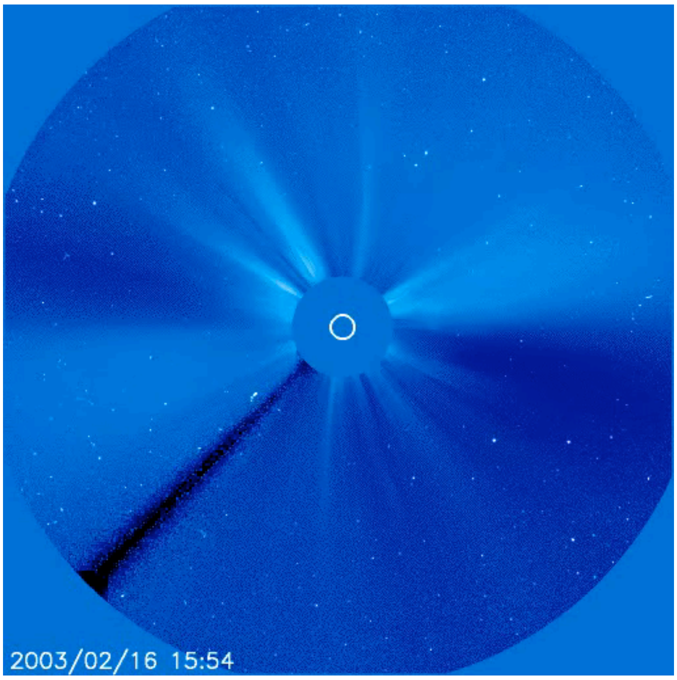
Lecture 3 - The Solar Wind

- Topics to be covered:
  - Parker's Solar Wind.
  - Inteplanetary Magnetic Field

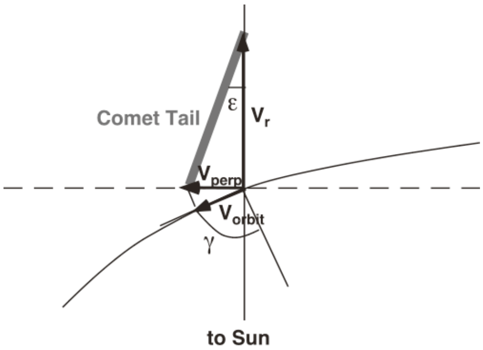


The Solar Wind

- Biermann (1951) noticed that many comets showed excess ionization and abrupt changes in the outflow of material in their tails - is this due to a solar wind?
- Assumed comet orbit perpendicular to line-of-sight (v_{perp}) and tail at angle ε =>

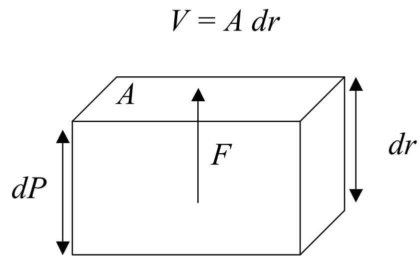
$$\tan\varepsilon = v_{perp}/v_r$$

- From observations, $\tan \varepsilon \sim 0.074$
- But v_{perp} is a projection of v_{orbit} => $v_{perp} = v_{orbit} \sin \gamma \sim 33\ km\ s^{-1}$
- From 600 comets, $v_r \sim 450\ km\ s^{-1}$.



Parker's Solar Wind

- Parker (1958) assumed that the outflow from the Sun is steady, spherically symmetric and isothermal.
- As $P_{Sun} >> P_{ISM}$ => must drive a flow.
- First consider static wind similar to Chapman (1957). The force across and given volume is
  $$F = dP\, A$$
- From Newton's 2nd law: $F = m\, a = \rho\, V\, a$.
  $$= \rho\, A\, dr\, a$$
- Therefore, $dP\, A = \rho\, A\, dr\, a$
- and, $\frac{dP}{dr} = \rho a$
- If $a = -G\, M_S / r^2$, we get $\boxed{\frac{dP}{dr} + \frac{GM_S \rho}{r^2} = 0}$ *Eqn 1.*
  Equation of Hydrostatic Equilibrium



Parker's Solar Wind (cont.)

- For a steadily expanding wind, $$\rho \frac{dv}{dt} = -\frac{dP}{dr} - \frac{GM_S \rho}{r^2} \qquad \textit{Eqn. 2}$$
  ($F = ma$)
- As, $\frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \frac{dv}{dr} v$ $\Rightarrow \rho v \frac{dv}{dr} = -\frac{dP}{dr} - \frac{GM_S \rho}{r^2}$
- or $$\boxed{v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM_S}{r^2} = 0} \qquad \textit{Eqn. 3}$$
- Called the *momentum equation*.
- *Eqn. 3* describes acceleration (1st term) of the gas due to a pressure gradient (2nd term) and gravity (3rd term). Would like *Eqn. 3* in terms of v.
- Assuming a perfect gas, $P = R\, \rho\, T / \mu$ (R is gas constant; μ is mean atomic weight), the 2nd term of *Eqn. 3* is:
  $$\frac{dP}{dr} = \frac{R\rho}{\mu}\frac{dT}{dr} + \frac{RT}{\mu}\frac{d\rho}{dr}$$
  Isothermal wind => $dT/dr \to 0$
  $$\Rightarrow \frac{1}{\rho}\frac{dP}{dr} = \left(\frac{RT}{\mu}\right)\frac{1}{\rho}\frac{d\rho}{dr} \qquad \textit{Eqn. 4}$$

Parker's Solar Wind (cont.)

- Now, the mass loss rate is assumed to be constant, so the *Equation of Mass Conservation* is: $\frac{dM}{dt} = 4\pi r^2 \rho v = const \Rightarrow r^2 \rho v = const$ *Eqn. 5*
- Differentiating, $\frac{d(r^2 \rho v)}{dr} = r^2 \rho \frac{dv}{dr} + \rho v \frac{dr^2}{dr} + r^2 v \frac{d\rho}{dr} = 0$

  $=> \frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{v}\frac{dv}{dr} - \frac{2}{r}$ *Eqn. 6*
- Substituting *Eqn. 6* into *Eqn. 4*, and into the 2nd term of *Eqn. 3*, we get

$$v\frac{dv}{dr} + \frac{RT}{\mu}\left(-\frac{1}{v}\frac{dv}{dr} - \frac{2}{r}\right) + \frac{GM_S}{r^2} = 0$$

$$\Rightarrow \left(v - \frac{RT}{\mu v}\right)\frac{dv}{dr} - \frac{2RT}{\mu r} + \frac{GM_S}{r^2} = 0$$

- A *critical point* occurs when *dv/dr* → *0* i.e., when $\frac{2RT}{\mu r} = \frac{GM_S}{r^2}$
- Setting $v_c = \sqrt{RT/\mu}$ => $r_c = GM_S / 2v_c^2$

Parker's Solar Wind (cont.)

- Rearranging => $\left(v^2 - v_c^2\right)\frac{1}{v}\frac{dv}{dr} = 2\frac{v_c^2}{r^2}(r - r_c)$ *Eqn. 7*
- Gives the momentum equation in terms of the flow velocity.
- If $r = r_c$, *dv/dr -> 0* or $v = v_c$, and if $v = v_c$, *dv/dr* -> ∞ or $r = r_c$.
- An acceptable solution is when $r = r_c$ and $v = v_c$ (*critical point*).
- An solution to *Eqn. 7* can be found by direct integration:

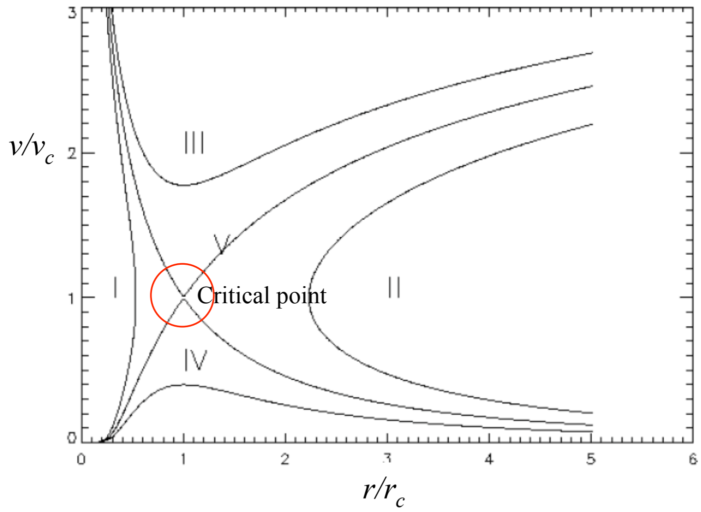
$$\left(\frac{v}{v_c}\right)^2 - \ln\left(\frac{v}{v_c}\right)^2 = 4\ln\left(\frac{r}{r_c}\right) + 4\frac{r_c}{r} + C$$

Eqn. 8
Parker's Solutions

where *C* is a constant of integration. Leads to five solutions depending on *C*.

Parker's Solutions

- Solution I and II are double valued. Solution II also doesn't connect to the solar surface.
- Solution III is too large (supersonic) close to the Sun - not observed.
- Solution IV is called the solar breeze solution.
- Solution V is the solar wind solution (confirmed in 1960 by Mariner II). It passes through the critical point at $r = r_c$ and $v = v_c$.



Parker's Solutions (cont.)

- Look at Solutions IV and V in more detail.
- Solution IV: For large r, $v \rightarrow 0$ and *Eqn. 8* reduces to:

$$-\ln\left(\frac{v}{v_c}\right)^2 \approx 4\ln\left(\frac{r}{r_c}\right) \Rightarrow v \approx \frac{1}{r^2}$$

- From *Eqn. 5:* $\rho = \frac{const}{r^2 v} \Rightarrow \rho \rightarrow \rho_\infty = const$
- From Ideal Gas Law: $P_\infty = R\,\rho_\infty\, T/\mu \Rightarrow P_\infty = const$
- The *solar breeze solution* results in high density and pressure at large r =>unphysical solution.

Parker's Solutions (cont.)

- Solution V: From the figure, $v >> v_S$ for large r. *Eqn. 8* can be written:

$$\left(\frac{v}{v_c}\right)^2 \approx 4\ln\left(\frac{r}{r_c}\right) \Rightarrow v \approx \left(\ln\left(\frac{r}{r_c}\right)\right)^{1/2}$$

- The density is then: $\rho = \frac{const}{r^2 v} \approx \frac{const}{r^2\sqrt{\ln(r/r_c)}}$

  => $\rho \rightarrow 0$ as $r \rightarrow \infty$.

- As plasma is isothermal (i.e., $T = const.$), Ideal Gas Law => $P \rightarrow 0$ as $r \rightarrow \infty$.

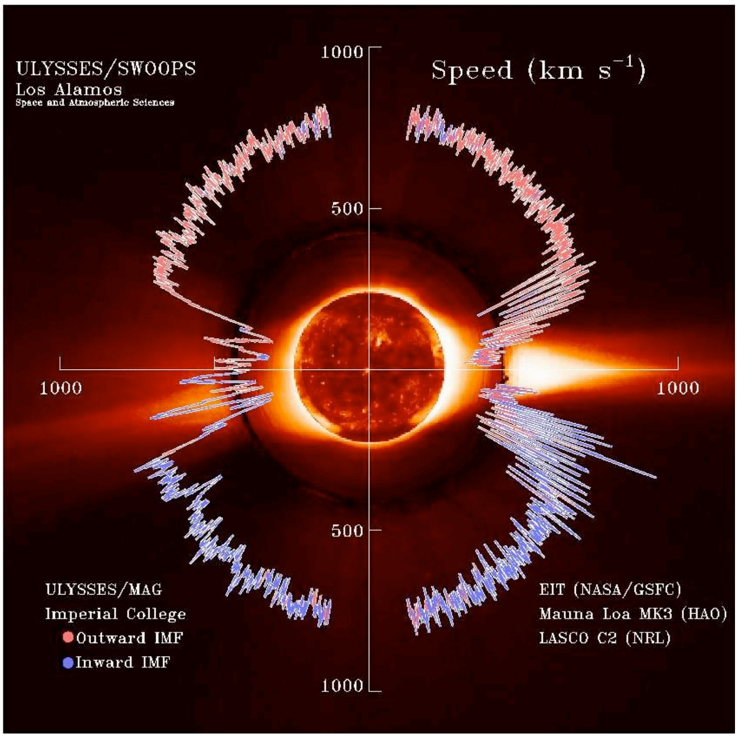
- This solution eventually matches interstellar gas properties => physically realistic model.

- Solution V is called the *solar wind solution*.

Observed Solar Wind

- Fast solar wind ($v \sim 700$ $km\ s^{-1}$) comes from coronal holes.

- Slow solar wind ($v < 500$ $km\ s^{-1}$) comes from closed magnetic field areas.



Interplanetary Magnetic Field

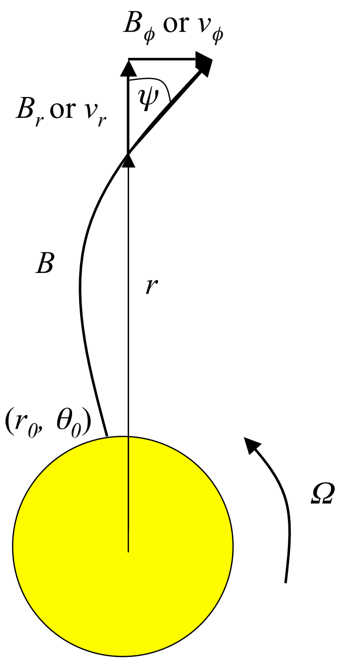
- Solar rotation drags magnetic field into an Archimedian spiral ($r = a\theta$).
- Predicted by Gene Parker => Parker Spiral:

  $$r - r_0 = -(v/\Omega)(\theta - \theta_0)$$

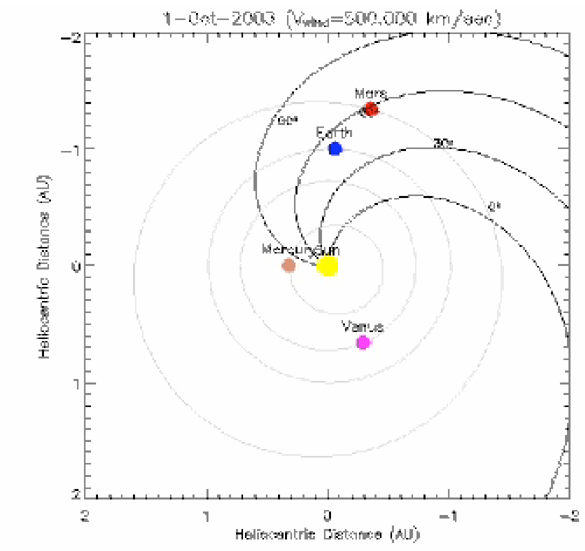
- Winding angle: $\tan\psi = \frac{B_\phi}{B_r} = \frac{v_\phi}{v_r}$

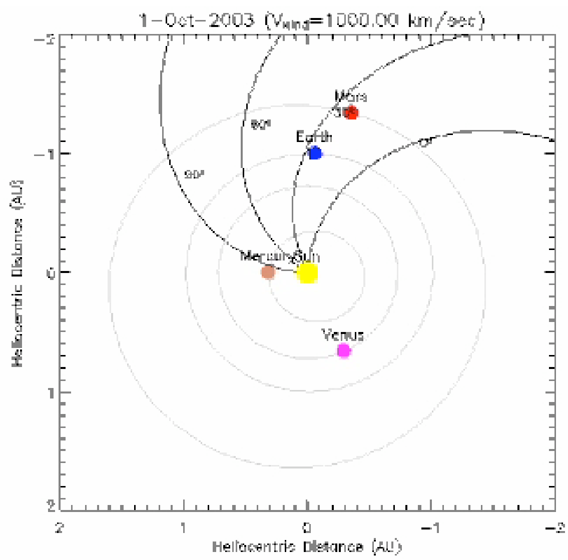
  $$= \frac{\Omega(r - r_0)}{v_r}$$

- Inclined at ~45º at 1 *AU* ~90º by 10 *AU*.



The Parker Spiral





- http://beauty.nascom.nasa.gov/~ptg/mars/movies/