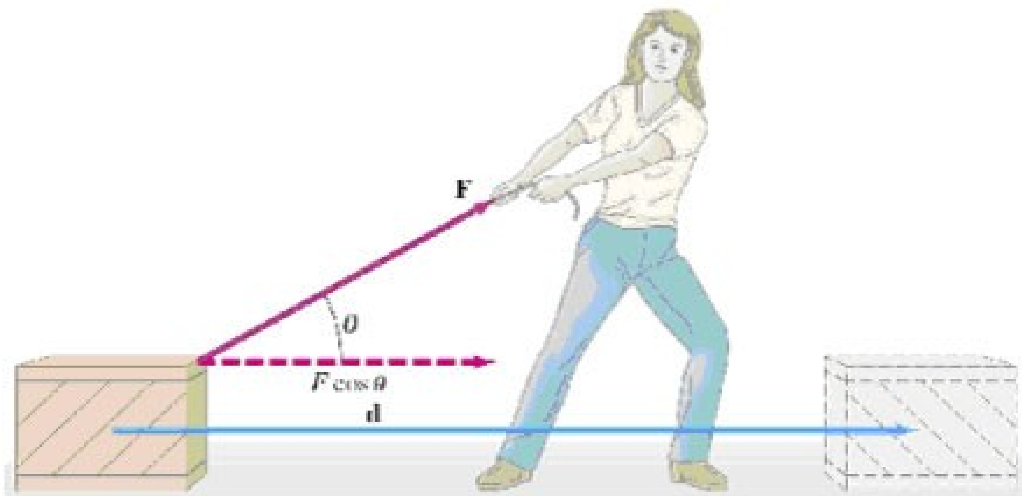
IL LAVORO



$$W = \vec{F} \cdot \vec{s} = F\cos(\vartheta)\, s = F_s s$$

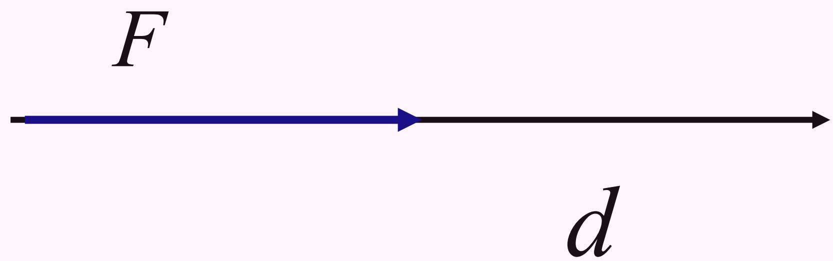
$$L = \vec{F} \cdot \vec{d} = F\, d \cos(\theta)$$

Ma la forza può cambiare nel tempo quindi:

$$d\,L = \vec{F} \cdot \vec{d}r$$

N.B.

$$\theta = 0 \quad \Longrightarrow \quad L = Fd\cos 0 = Fd$$



Se $F = 1\ N$ e $d = 1\ m$

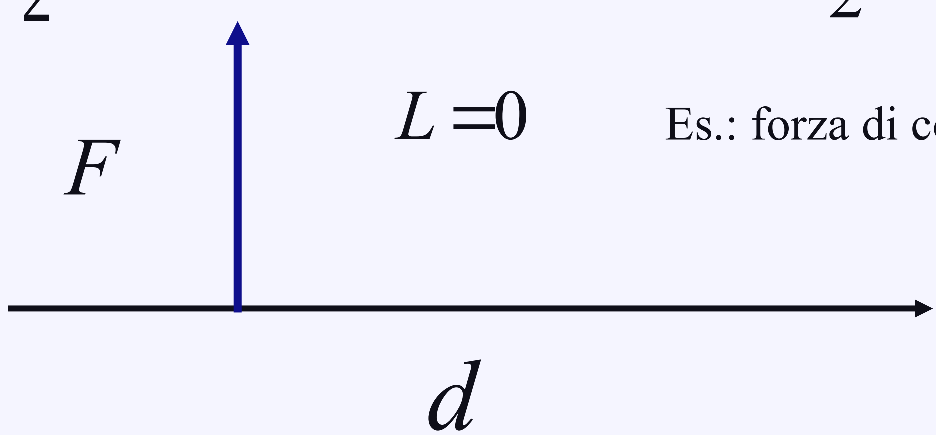
$$L = 1\ N \times 1\ m = 1\ J(oule)$$

unità di misura del lavoro.

$$\theta = \frac{\pi}{2} \quad \Longrightarrow \quad L = Fd\cos\frac{\pi}{2} = 0$$

$$L = 0$$

Es.: forza di contatto



Esempio: CAMPO MAGNETICO!

Energia cinetica

$$dL = \vec{F} \cdot d\vec{r} = m\,\vec{a} \cdot d\vec{r} = m\,\frac{d\,\vec{v}}{dt}\,d\vec{r}$$

$$dL = m\,a\,dx = m\,\frac{d\,v}{dt}\,dx$$ **In 1 Dim**

Il lavoro totale fatto lungo un percorso sarà

$$L = \lim_{N \to \infty} \sum_{i=1}^{N} m\,\frac{dv_i}{dt}\,dx_i = \lim_{N \to \infty} \sum_{i=1}^{N} m\,\frac{dv_i}{dt}\,v_i\,dt$$

$$L = \lim_{N \to \infty} \sum_{i=1}^{N} m\,dv_i\,v_i$$

$$L = \int_{v_1}^{v_2} m\,v\,dv = \left[\frac{1}{2}m\,v^2\right]_{v_1}^{v_2} = \frac{1}{2}m\,v_2^2 - \frac{1}{2}m\,v_1^2$$

Energia cinetica

$$E_K = \frac{1}{2}m\,v^2$$

$$L = \Delta E_K$$

$$J = 10^7\,erg$$

Energia cinetica-2 (formale)

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} \quad cambio \;\; di \;\; variabile$$

$$L = \int_{x_1}^{x_2} m\, a\, dx = \int_{x_1}^{x_2} m\, v \frac{dv}{dx} dx$$

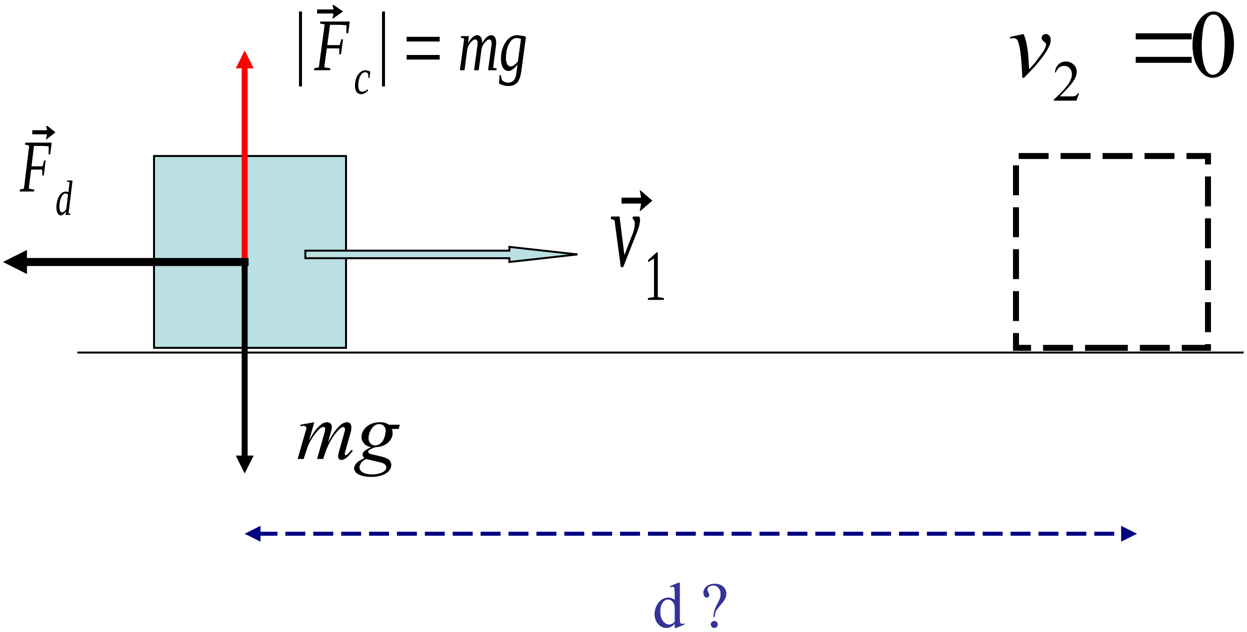
Si integra per sostituzione:

$$\int_{x_1}^{x_2} f\left(g\left(x\right)\right) g'\left(x\right) dx = \int_{g(x_1)}^{g(x_2)} f\left(x\right) dx$$

Allora, assumendo che v(x) = g(x)

$$L = \int_{v_1}^{v_2} m\, v\, dv = \frac{1}{2} m\, v_2^2 - \frac{1}{2} m\, v_1^2$$

Esempio: calcolo della distanza di arresto per attrito



$$L = \vec{F}_d \cdot \vec{d} = \mu_d mgd \cos\pi = -\mu_d mgd$$

$$L = -\mu_d mgd = \frac{1}{2} m 0^2 - \frac{1}{2} m v_1^2$$

$$\mu_d \cancel{m} gd = \frac{1}{2} \cancel{m} v_1^2$$

$$\boxed{d = \frac{v_1^2}{2\mu_d g}}$$

Potenza

$$\bar{P} = \frac{L}{\Delta t}$$ Potenza media

$$P = \frac{dL}{dt}$$ Potenza istantanea

$$P = \frac{1\,J}{1\,s} = 1\,W \qquad \text{(Watt)}$$