Introduction to QED: II Partial Exam – 11/06/2014

1 Decay of massive vector boson

Let us consider the following Lagrangian describing the interaction between two Dirac fields $\psi_{1,2}$ of mass $m_{1,2}$ and a massive charged vector boson W_μ of mass M:

$$\mathcal{L} \;=\; -\frac{1}{2} W^\dagger_{\mu\nu} W^{\mu\nu} \,+\, M^2 W^\dagger_\mu W^\mu \,+\, \sum_i \bar{\psi}_i \left(i\partial\!\!\!/ - m_i \right) \psi_i \,+\, \mathcal{L}_{int}$$

where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ is the massive vector field strength and the interaction Lagrangian is given by

$$\mathcal{L}_{int} \;=\; \bar{\psi}_1 \gamma^\mu \left(g_L P_L + g_R P_R \right) \psi_2 \, W_\mu \,+\, \text{h.c.}$$

with $P_{R,L} = (1 \pm \gamma_5)/2$ the usual chiral projectors and g_L, g_R real numbers. Let furthermore remind that a massive vector boson field, in terms of the creation and annihilation operators, read:

$$W_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2\omega_k}} \sum_{\lambda=1}^{3} \left[a_{(\lambda)}(k)\, e^{-ik\cdot x} + b^\dagger_{(\lambda)}(k)\, e^{ik\cdot x} \right] \epsilon^{(\lambda)}_\mu(k) \Bigg|_{k_0=\omega_k}$$

with $\omega_k = \sqrt{M^2 + |\vec{k}|^2}$.

1. Derive explicitly the expression for the hermitian conjugate term (i.e. h.c.) in the interaction Lagrangian $\mathcal{L}_{int}$;
2. Calculate the equation of motion for the charged vector field W_μ;
3. Defining respectively q_W, q_1 and q_2 the charges of the vector and fermionic fields, show that for appropriate values of q_i the full Lagrangian has a global $U(1)$ symmetry. Explain why the Lagrangian cannot possess, instead, a local $U(1)$ symmetry;
4. Derive explicitly the Feynman rule associated to the vector-fermion interaction (i.e. choose a specific process and calculate the associated first order Feynman amplitude);

From now on assume that the vector particle W is at rest in the Laboratory frame. Assume also $M > 2\, m_1$ and $m_2 = 0$.

5. Calculate the unpolarized differential decay-rate for the decay process $W \to l_1 + \bar{l}_2$ (with l_1 we mean the particle of type 1 and with $\bar{l}_2$ the antiparticle of type 2) in the Lab. frame;
6. Calculate the polarized differential decay-rate for the decay process $W \to (l_1)_L + (\bar{l}_2)_R$ (with the particle 1 with helicity $-1/2$ and the antiparticle 2 with helicity $+1/2$) in the Lab. frame;
7. Discuss briefly the polarized differential decay-rate for the decay process $W \to (l_1)_L + (\bar{l}_2)_L$.

All results obtained in class must be adequately discussed and motivated.

2 Parity transformation for Dirac fermions

Parity transformation for a Dirac field is defined as

$$\mathcal{U}_P^\dagger\, \psi(x,t)\, \mathcal{U}_P = S_P\, \psi(-x,t) \qquad , \qquad \mathcal{U}_P^\dagger\, \bar{\psi}(x,t)\, \mathcal{U}_P = \bar{\psi}(-x,t)\, S_P^\dagger$$

with $S_P = \eta_P \gamma_0$, and $\eta_P^2 = \pm 1$.

1. Show that the following relations for Dirac spinors hold:

$$\gamma_0 u_r(\vec{k}) = u_r(-\vec{k}) \qquad , \qquad \gamma_0 v_r(\vec{k}) = -v_r(-\vec{k})\,.$$

2. Using previous relations show that these definitions are equivalent to the following transformations on the annihilation operators:

$$\begin{aligned} \mathcal{U}_P^\dagger\, c_r(\vec{k})\, \mathcal{U}_P &= \eta_P\, c_r(-\vec{k}) \\ \mathcal{U}_P^\dagger\, d_r(\vec{k})\, \mathcal{U}_P &= -\eta_P^*\, d_r(-\vec{k})\,. \end{aligned}$$

3. Given the explicit realization of $\mathcal{U}_P$:

$$\mathcal{U}_P = exp\left\{ i\frac{\pi}{2}\int d^3k \sum_r \left[\left(c_r^\dagger(\vec{k})c_r(\vec{k}) - d_r^\dagger(\vec{k})d_r(\vec{k}) \right) - \eta_P \left(c_r^\dagger(\vec{k})c_r(-\vec{k}) + d_r^\dagger(\vec{k})d_r(-\vec{k}) \right) \right] \right\}$$

show that the Hamiltonian of the free Dirac theory is left invariant by a parity transformation.

All results obtained in class must be adequately discussed and motivated.