Dipartimento di Fisica e Astronomia “Galileo Galilei” - Università di Padova

Introduction to Quantum ElectroDynamics

Dott. Stefano Rigolin
Office 305 - Via Marzolo 8

June 15, 2014

Detailed Lecture Plan

[05/03] – Presentation of the course (1h) – Administrative information, brief summary of prerequisites, tentative program and goals;

– General Motivations (1h) – Why the need of a relativistic QFT: multi-particle states and causality;

[06/03] – System with a finite number of degrees of freedom (1h). Lagrangian vs Hamiltonian formalism: principle of Least Action and Eulero–Lagrange equations of motion. Hamilton equations as Poisson parenthesis;

– System with an infinite number of degrees of freedom (1h). Lagrangian vs Hamiltonian formalism for fields: principle of Least Action and Eulero–Lagrange equations of motion. Hamilton equations as Poisson parenthesis;

[07/03] – Functional and Functional derivatives (1h) – Definition of Functional and Functional derivative. Hamilton equations as Poisson parenthesis;

[12/03] – Symmetries and Noether theorem (2h). Spacetime symmetries and conservation of momentum and angular momentum. Internal symmetries and charge conservation;

[13/03] – (Classical) Real Scalar Field Theory (1h) – Lagrangian for real scalar fields. Eulero-Lagrange equations and spacetime conserved currents. Hamiltonian formalism and Poisson parenthesis;

– Complex Scalar Field Theory (1h) – Lagrangian for complex scalar fields. Eulero-Lagrange equations, spacetime and internal conserved currents. Hamiltonian formalism and Poisson parenthesis.

[14/03] – General Solution of Klein-Gordon equation (1h) – Explicit derivation for complex and real scalar fields.

[26/03] – Canonical Quantization for real scalar fields (2h) – Conserved operators in the momentum space. Normal ordering. Definition of Fock space of particle states. Normalization of states. Spin-statistics connection. Covariant commutators

[27/03] – Canonical Quantization for complex scalar fields (1h) – Extension to the complex scalar field case. Internal global $U(1)$ symmetry and definition of particle charge;

– Summary of Dirac equation (1h). Review of the formalism. Dirac matrices in the Dirac and Weyl representation. Covariance of the Dirac equation and Lorentz generators in the spinorial space. Dirac conjugate spinor.

[28/03] – General solution of the Dirac equation (1h). Plane wave solutions of the Dirac equation. Positive and negative energy spinors in the momentum space. Explicit expressions for the solutions with the Dirac representation of the γ matrices. Positive and negative energy projectors. Polarization projectors. Helicity in the $E \gg m$ limit and chirality.

[02/04] – Fermion Field Theory (1h). Lagrangian for Dirac fermions. Eulero-Lagrange equations and spacetime conserved curents. Hamiltonian formalism and Poisson parenthesis;

– Canonical Quantization of the Dirac field (1h). Quantization with commutators and inconsistencies. Quantization with anticommutators. Fock space for fermions. Connection with spin-statistics theorem.

[03/04] – Covariant anticommutators and causality (1h). Covariant anticommutators for Dirac fields and causality. Transformation properties of Dirac matrices and fermionic bilinears structures;

– Classical vector field theory (1h). Massive vector field Lagrangian theory. The Proca Lagrangian. Eulero-Lagrange equation for a massive vector field. Poincare group invariance and conserved currents. Hamiltonian formalism and canonical quantization (outline);

[04/04] – Classical Electromagnetic field theory (1h). Maxwell equations and Electromagnetic Lagrangian. Gauge Invariance. Coulomb (radiation) gauge. Solution of the Eulero-Lagrange equation in the Coulomb gauge. The Lorentz gauge;

[09/04] – Gauge fixing Lagrangian (1h). Modification of the Electromagnetic Lagrangian and gauge fixing term (ξ gauge). Lagrangian and Hamiltonian densities in the Feynman gauge ($\xi = 1$). General solution of the equation of motion for $\xi = 1$.

– Covariant quantization of the E.M. field (1h). Canonical quantization in the Feynman gauge. Commutation relations for fields and creation/annihilation operators. Explicit expressions for the Hamiltonian operator. Fock space and indefinite metric. Unphysical polarization and negative norm/energy states. Gupta-Bleuler condition. Properties of the physical (photon) and unphysical (pseudo-photon) states. Covariant commutators and causality;

[10/04] – Interactions in classical field theory (1h). Examples of interactions in classical field theory. Scalar self-interaction, Yukawa interaction and gauge interaction. Renormalizability condition (and power counting);

– Gauge theories (1h). Minimal coupling and gauge invariance. Covariant derivative. QED and Scalar QED Lagrangians;

[11/04] – Dirac equation with E.M. interaction (1h). Dirac equation for the interaction of a charged fermion with the E.M. field. Pauli equations and non–relativistic limit of Dirac equation. Gyromagnetic moment of the electron;

[16/04] – Interactions in QFT (1h). Obstacles in the canonical quantization of an interacting QFT. The Schrodinger, Heisemberg and Interaction pictures. Evolution of operators and states in the interaction picture.

– Interaction picture (1h). The evolution operator in the Interaction picture. Perturbative expansion of the evolution matrix for weak interacting theories;

[17/04] – Definition of the S-matrix (1h). Definition of the S-matrix for asymptotic states. Porperties of the S-matrix;

– Wick theorem (1h). Definition of T-product and Feynman propagator for real and complex scalar fields. Wick theorem for scalar fields.

[30/04] – First partial written test (2h).

[07/05] – Wick theorem (1h). Corollary of Wick theorem for N-product of fileds at the same point. Extension of Wick theorem for vectorial and spinorial fields;

– Feynman propagators (1h). Deeper look to Feynman propagators. Physical meaning and diagrammatic representation. Explicit derivation of the scalar, vector and spinor propagators;

[08/05] – S-matrix expansion in QED (2h). Explicit derivation of first terms of the S-matrix expansion. Interpretation of first order terms in the expansion;

– Interpretation of second order terms in the expansion. Diagrams in coordinate space with 0 or 1 propagator.

[09/05] – Tutorship on gamma matrices and traces of gamma matrices (1h).

[14/05] – Feynman diagrams in coordinate space (2h). Feynman diagram in coordinate space for Compton scattering and Bhabha scattering;

– Diagrams in coordinate space with 2 propagators. Photon and fermion self-energy diagrams;

[15/05] – Feynman diagrams in momentum space (2h). Calculation of matrix elements between states of definite momentum. Propagators and field contractions. 0th order processes in momentum space. 1st order processes in momentum space and Compton scattering. Loops in momentum space. Summary of QED Feynman rules;

[16/05] – Decay rate (1h). Study of $1 \to 2$ decay. Two body phase space and derivation of differential and total decay width in the Laboratory frame;

[21/05] – Cross section (1h). Study of $2 \to 2$ scattering process. Two body phase space and derivation of differential and total cross section in the Center of Mass frame;

– Elementary processes in QED (1h). Explicit derivation of the $e^- + e^+ \to \mu^- + \mu^+$ un-polarized cross section;

[22/05] – Elementary processes in QED (2h). High energy limit of the $e^- + e^+ \to \mu^- + \mu^+$ scattering. Chirality vs Helicity. Polarized cross section for $e^- + e^+ \to \mu^- + \mu^+$;

– Crossing symmetry in the high energy limit. Derivation of the $e^- + \mu^- \to e^- + \mu^-$ cross section formula;

[23/05] – Compton scattering (1h). Feynman diagrams for the Compton scattering. Sum over photon polarizations and Ward identities;

[28/05] – Compton scattering (1h). Differential cross section in Lab and CM frame. Low energy and high energy limits. Mass divergence;

– Massive scalar boson decay in fermions and Yukawa theory Feynman rules (1h).

[29/05] – Massive vector boson decay in fermions (1h).

– Discrete symmetries (1h). Parity transformation for scalars, vectors and fermions;

[30/05] – Discrete symmetries (2h). Time reversal and charge comjugation for scalar, vectors and fermions. CPT theorem;