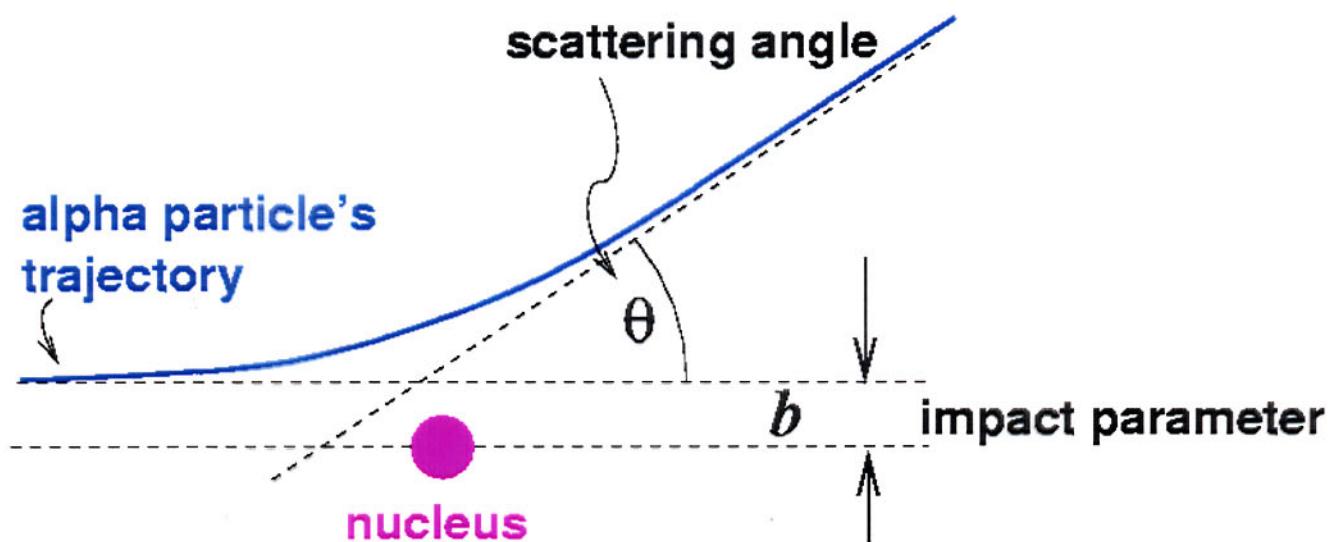




Fisica 5F AA 2007/2008

L'esperimento di Rutherford (1911)



La distribuzione dell'angolo di diffusione

$$N(\Theta) = \frac{N_i \cdot n \cdot t \cdot Z^2 \cdot e^4}{(8\pi\varepsilon_0)^2 \cdot r^2 \cdot E_\alpha^k \cdot \sin^4(\theta/2)}$$

N_i=numero totale di particella alfa

n = numero di atomi per unità di volume

t = spessore del bersaglio

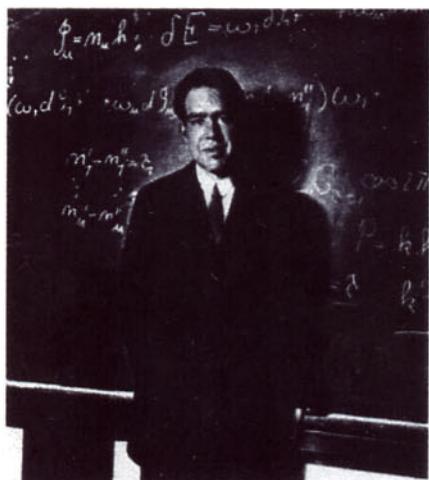
E_k = energia cinetica della particella

Z_e = carica elettrica del nucleo



Fisica 5F AA 2007/2008

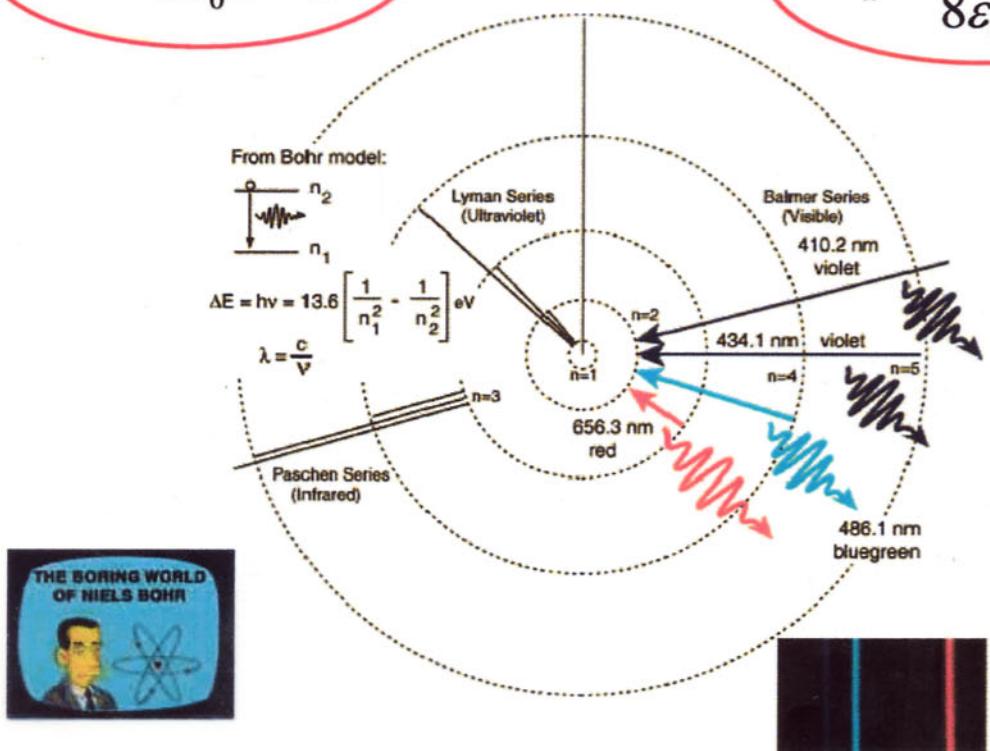
Lo spettro dell'idrogeno



1. Gli elettroni atomici occupano solamente delle orbite particolari o stati stazionari
2. L'energia viene persa o acquisita mediante l'assorbimento o la cessione di un fotone di energia $h\nu$, che corrisponde alla differenza di energia tra i due stati stazionari
3. La condizione di quantizzazione riguarda il momento della quantità di moto $L=nh/2\pi$
4. Non serve la meccanica relativistica

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2} \cdot \frac{1}{n^2}$$

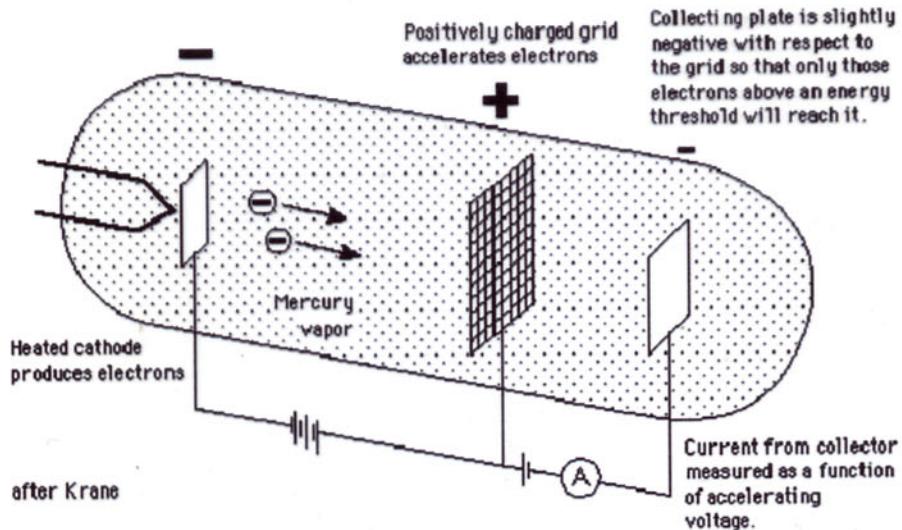
$$R_x = \frac{me^4}{8\varepsilon_0^2 h^3 c}$$



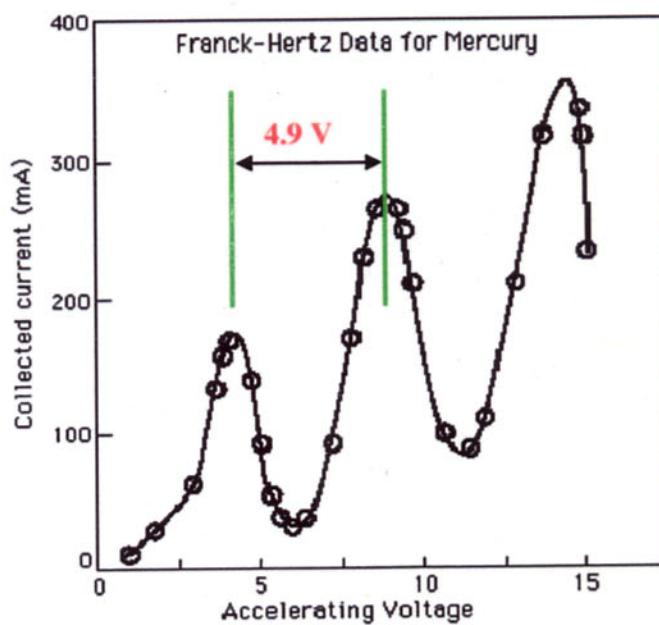


Fisica 5F AA 2007/2008

L'esperimento di Franck-Hertz



La distanza caratteristica di 4.9 V tra massimi successivi corrisponde ad una lunghezza d'onda di 254 nm



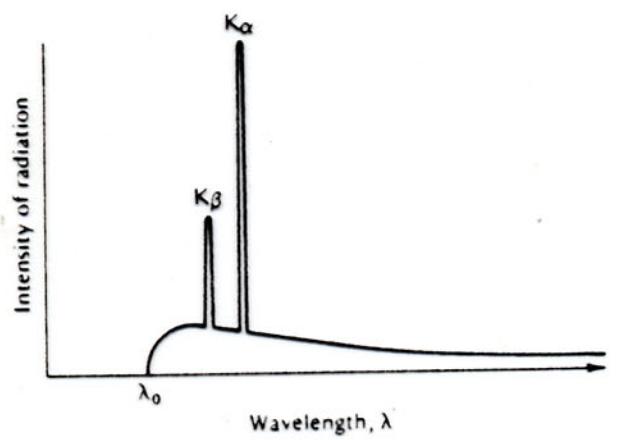


Fig. 3. Spectrum of X-rays produced when a cathode-ray beam strikes a target made of a single element.

Table I
Wavelengths of the K_α and K_β Radiation, as Reported by Moseley

Element	λ_{α} (Å)	λ_{β} (Å)	$\sqrt{(\nu_{\alpha}/\frac{3}{4}\nu_0)^4}$	Z
Ca	3.357	3.085	19.00	20
Ti	2.766	2.528	20.99	22
V	2.521	2.302	21.96	23
Cr	2.295	2.088	22.98	24
Mn	2.117	1.923	23.99	25
Fe	1.945	1.765	24.99	26
Co	1.796	1.635	26.00	27
Ni	1.664	1.504	27.04	28
Cu	1.548	1.403	28.01	29
Zn	1.446	—	29.01	30

See text for explanation of this quantity.



100 eV?

 \sqrt{V}

cometrical
very large
ision of an
length of
ical optics
are always
negligible
which is

becomes
it, we are
tion angle
the wave
cts in the
f suitable
mentalists
of atoms
mensions
e 3-1, we
baseball,
ller mass
 $\lambda = h/p$
atus with
 $= 1.2 \text{ \AA}$

might be
unely by
olid. The
s for the
acteristic
nents by

Electrons
t emerge
t normal
ar angle
es of the
scattered
ik in the
Broglie's
f waves
al. The
' which
nnot be

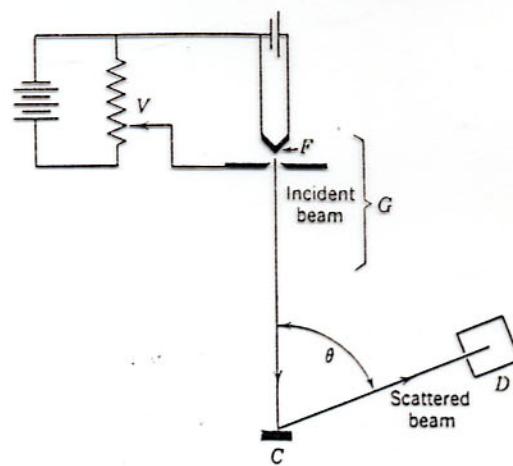


FIGURE 3-1

The apparatus of Davisson and Germer. Electrons from filament F are accelerated by a variable potential difference V . After scattering from crystal C they are collected by detector D .

understood on the basis of classical particle motion, but only on the basis of wave motion. Classical particles cannot exhibit interference, but waves can! The interference involved here is not between waves associated with one electron and waves associated with another. Instead, it is an interference between different parts of the wave associated with a single electron that have been scattered from various regions of the crystal. This can be demonstrated by using an electron beam of such low intensity that the electrons go through the apparatus one at a time, and by showing that the pattern of the scattered electrons remains the same.

Figure 3-3 shows the origin of a Bragg reflection, obeying the *Bragg relation* derived in the caption to that figure

$$n\lambda = 2d \sin \varphi \quad (3-3)$$

For the conditions of Figure 3-3 the effective interplanar spacing d can be shown by x-ray scattering from the same crystal to be 0.91 \AA . Since $\theta = 50^\circ$, it follows that $\varphi = 90^\circ - 50^\circ/2 = 65^\circ$. The wavelength calculated from (3-3), assuming $n = 1$, is

$$\lambda = 2d \sin \varphi = 2 \times 0.91 \text{ \AA} \times \sin 65^\circ = 1.65 \text{ \AA}$$

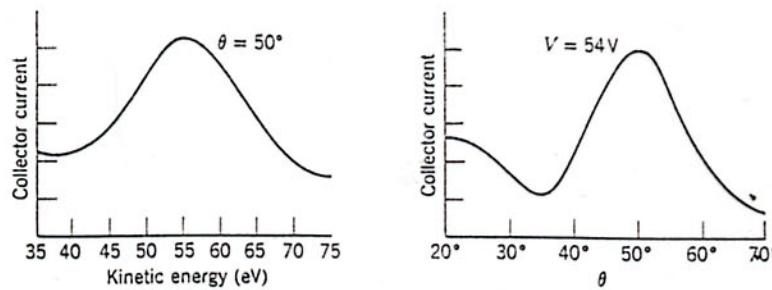


FIGURE 3-2

Left: The collector current in detector D of Figure 3-1 as a function of the kinetic energy of the incident electrons, showing a diffraction maximum. The angle θ in Figure 3-1 is adjusted to 50° . If an appreciably smaller or larger value is used, the diffraction maximum disappears. Right: The current as a function of detector angle for the fixed value of electron kinetic energy 54 eV.



Fisica 5F AA 2002/03

Proprietà ondulatorie della materia

Caratteristiche di fenomeni ondulatori stazionari

Elettroni in un'orbita di Bohr

Electron wave resonance

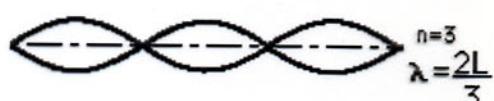
$$\text{Wavy line symbol with } n=1 \text{ above it}$$

$$\text{Wavy line symbol with } n=2 \text{ above it}$$

$$\text{Wavy line symbol with } n=3 \text{ above it}$$

Onde stazionarie in una corda tesa

String resonance modes



Visualizzazione del fenomeno ondulatorio di elettroni in un'orbita di Bohr

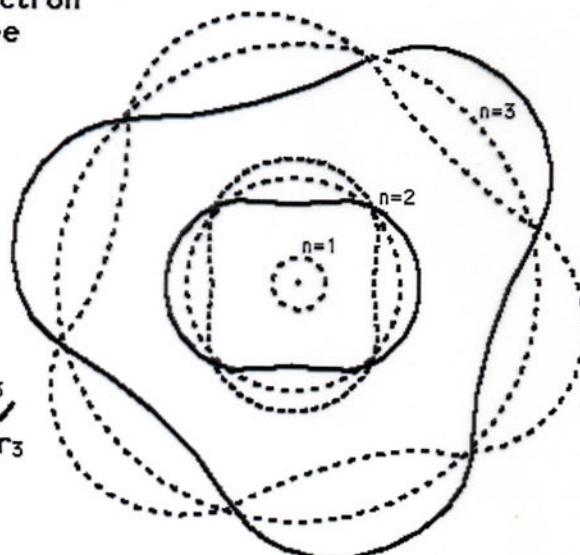
Visualization of electron waves for first three Bohr orbits

Electron wave resonance

$$\text{Wavy line symbol with } n=1 \text{ above it}$$

$$\text{Wavy line symbol with } n=2 \text{ above it}$$

$$\text{Wavy line symbol with } n=3 \text{ above it}$$



120 WAVES AND PARTICLES

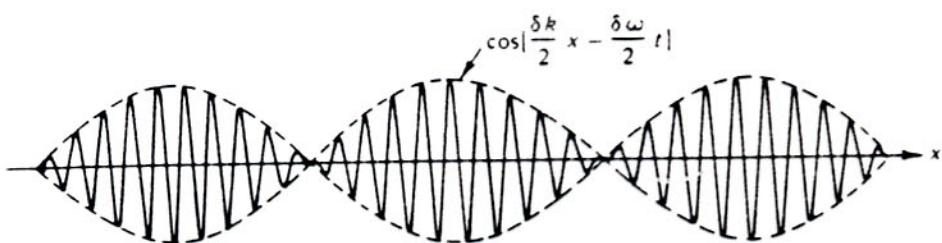


Fig. 10. Sum of two sine waves, of equal amplitude, differing in frequency by $\delta\omega$. The sum oscillates between $+2 \cos[(\delta k/2)x - (\delta\omega/2)t]$ and $-2 \cos[(\delta k/2)x - (\delta\omega/2)t]$, shown by the broken lines. As time passes this envelope moves with a speed of $\delta\omega/\delta k$, while the crests within the envelope move with speed ω/k .



Fisica 5F AA 2002/03

Proprietà ondulatorie della materia

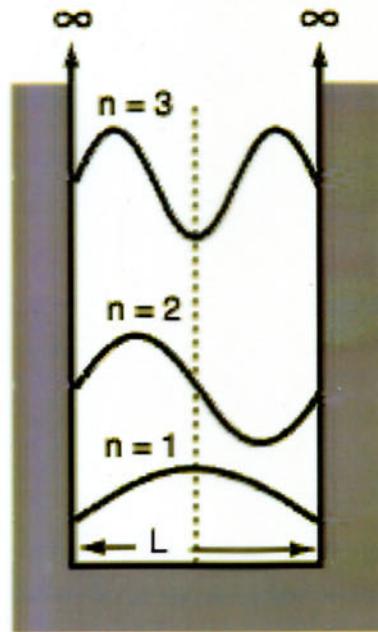
Una particella in una buca di potenziale molto profonda

l'equazione di Schrödinger
 $d^2\psi/dx^2 + 2m/(h / 2\pi)^2 \cdot E \cdot \psi = 0$

la funzione d'onda
 $\Psi(x) = 2/L \cdot \sin(n\pi/L \cdot x)$

la quantità di moto degli stati stazionari
 $p_n = n \hbar / 2L$

le energie degli stati stazionari
 $E_n = n^2 \hbar^2 / 8mL^2$

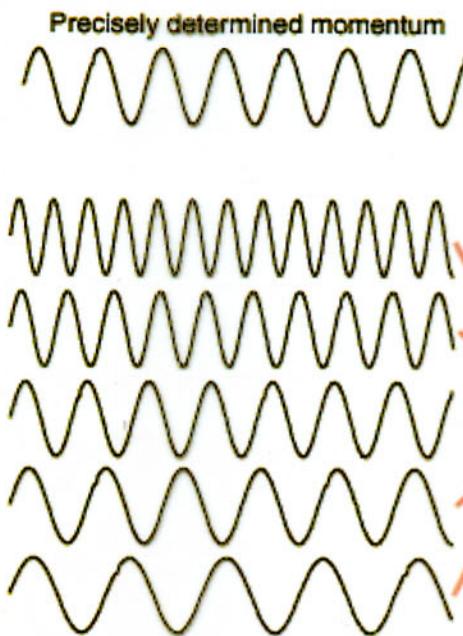


$x = 0$ at left wall of box.



Fisica 5F AA 2002/03

Il principio di indeterminazione di Heisenberg (1927) e i pacchetti d'onda

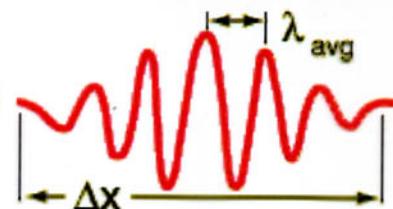


Precisely determined momentum
A sine wave of wavelength λ implies that the momentum p is precisely known:
But the wavefunction and the probability of finding the particle $\psi^*\psi$ is spread over all of space.

$$p = \frac{h}{\lambda}$$

p precise
x unknown

Adding several waves of different wavelength together will produce an interference pattern which begins to localize the wave.



but that process spreads the momentum values and makes it more uncertain. This is an inherent and inescapable increase in the uncertainty Δp when Δx is decreased.

$$\Delta x \Delta p > \frac{\hbar}{2}$$

5-6 Required Properties of Eigenfunctions

In the following section we shall consider, in a very general way, the problem of finding solutions to the time-independent Schroedinger equation. These considerations will show that energy quantization appears quite naturally in the Schroedinger theory. We shall see that this extremely significant property results from the fact that *acceptable solutions* to the time-independent Schroedinger equation can be found only for certain values of the total energy E .

To be an acceptable solution, an eigenfunction $\psi(x)$ and its derivative $d\psi(x)/dx$ are required to have the following properties:

- | | |
|--|--|
| $\psi(x)$ must be <i>finite</i> . | $d\psi(x)/dx$ must be <i>finite</i> . |
| $\psi(x)$ must be <i>single valued</i> . | $d\psi(x)/dx$ must be <i>single valued</i> . |
| $\psi(x)$ must be <i>continuous</i> . | $d\psi(x)/dx$ must be <i>continuous</i> . |

These requirements are imposed in order to ensure that the eigenfunction be a mathematically "well-behaved" function so that measurable quantities which can be evaluated from the eigenfunction will also be well-behaved. Figure 5-8 illustrates the

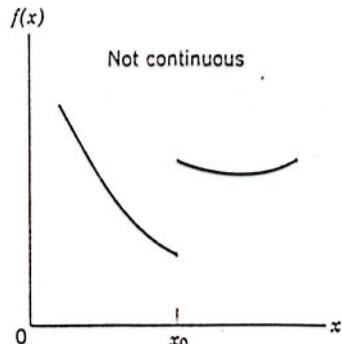
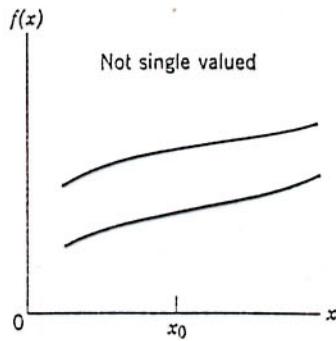
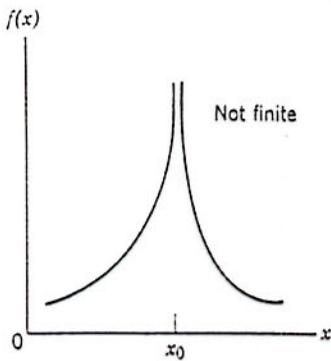


FIGURE 5-8

Illustrating functions which are not finite, not single valued, or not continuous, at a point x_0 .

to

ie

THIS FIGURE IS CONSISTENT WITH THE PROBABILITY DENSITY OF $|\psi_1|^2 = 0$ AT $x = L$

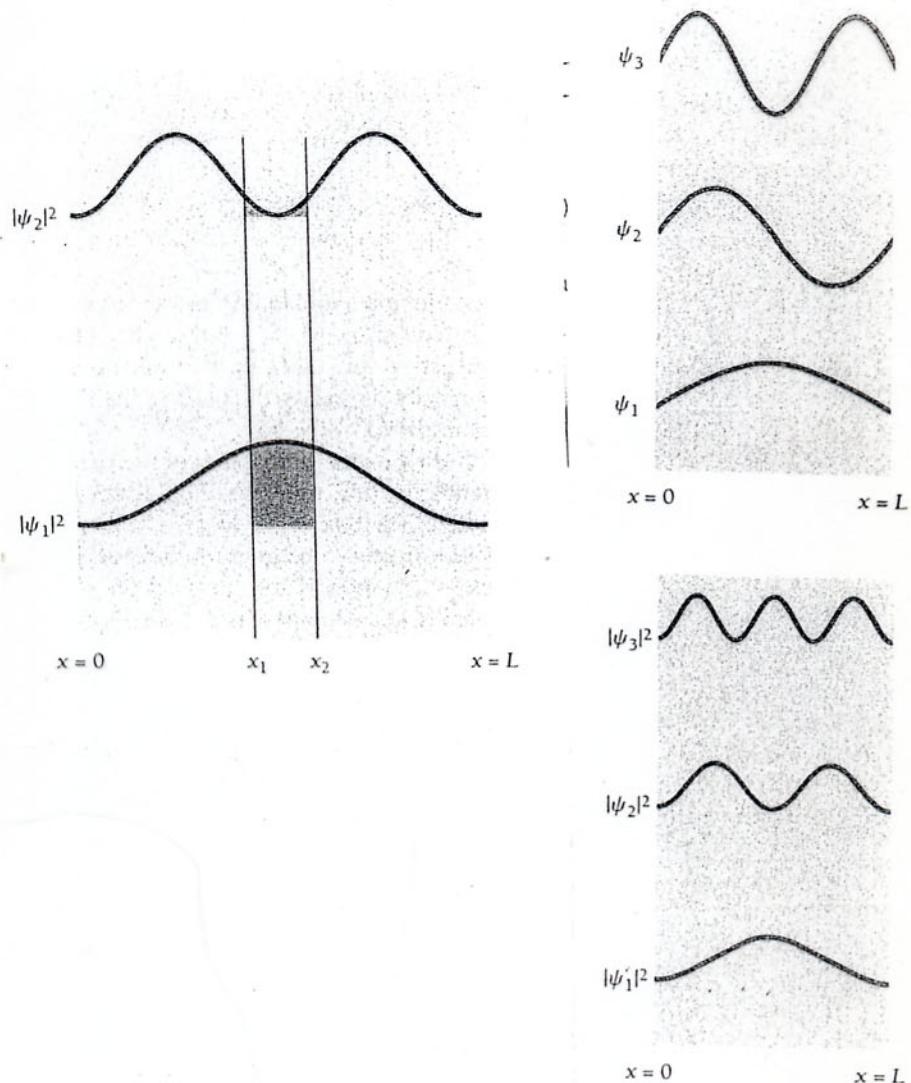


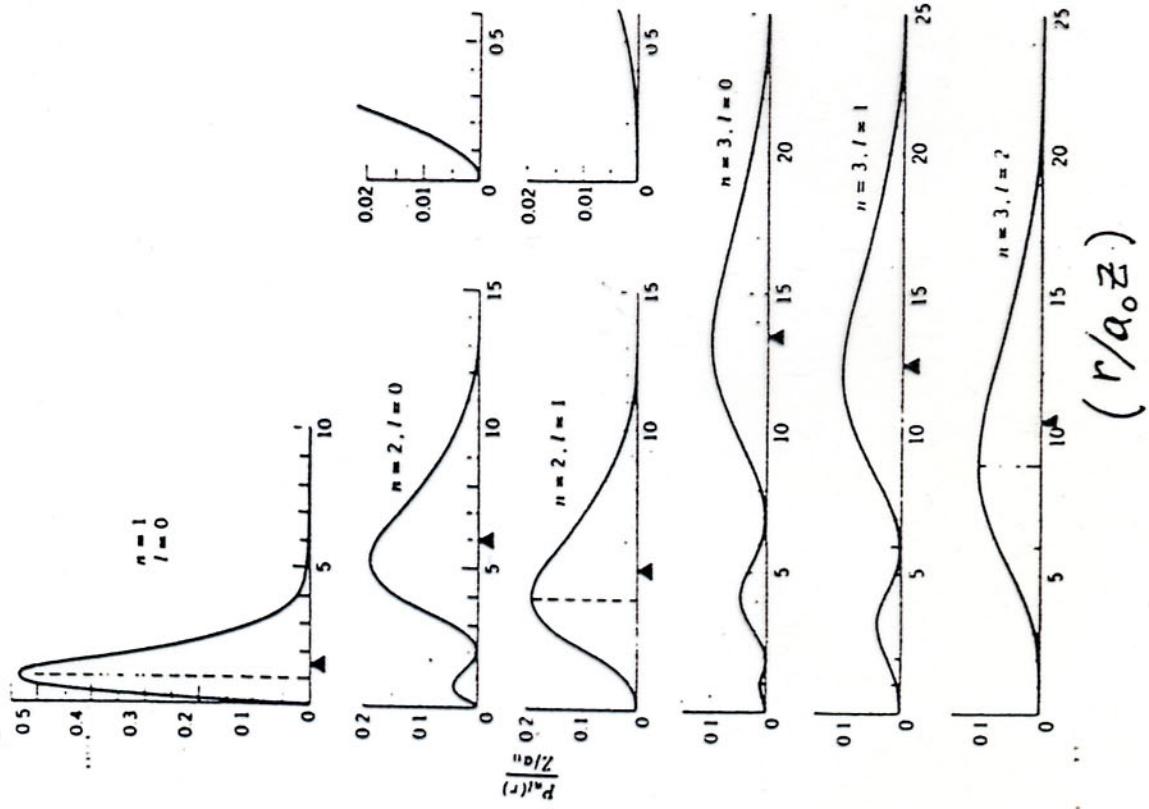
Figure 5.5 Wave functions and probability densities of a particle confined to a box with rigid walls.

IDROGENOIDI

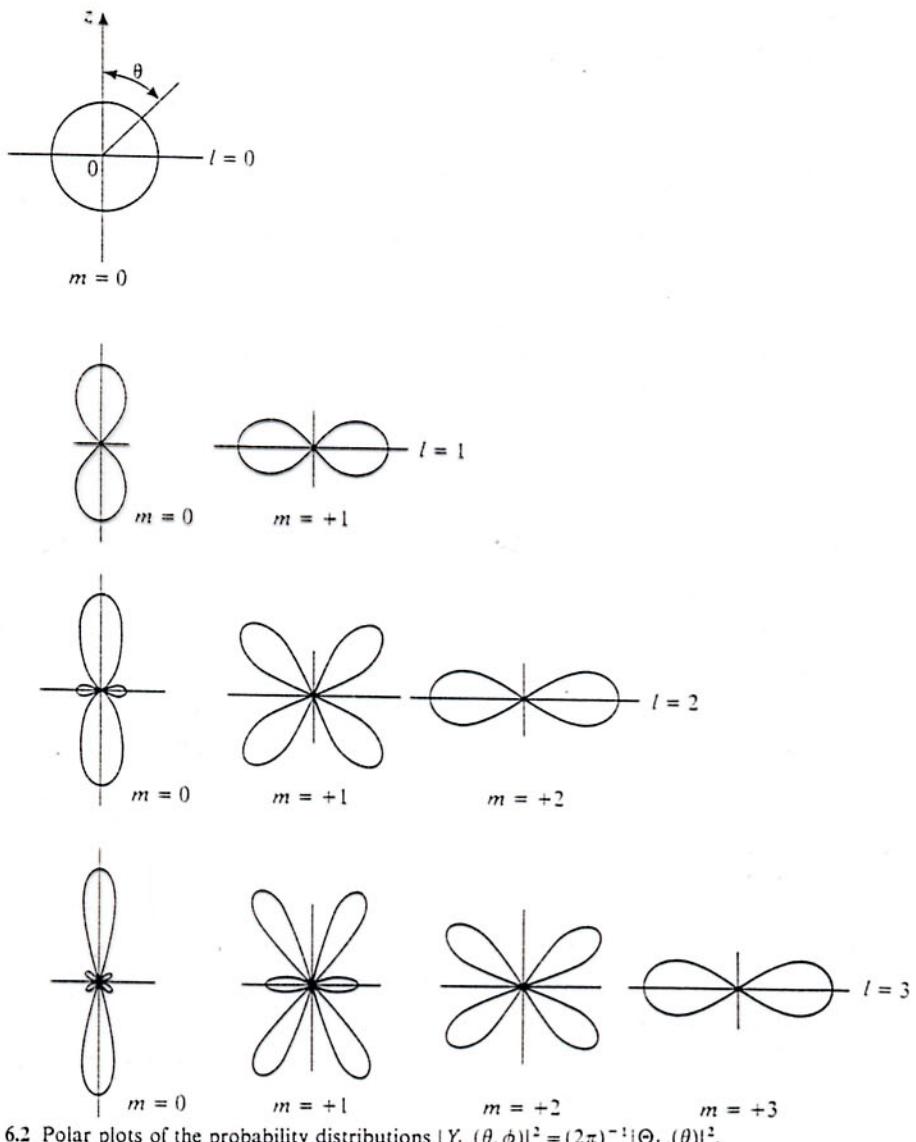
AUTOFUNZIONI PER ATOMI IDROGENOIDI

n	l	m_l	
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(2 - \frac{Z^2}{a_0^2}\right) e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \cos \theta$
2	1	± 1	$\psi_{21\pm 1} = \frac{1}{8\sqrt{3}\pi} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \sin \theta e^{\pm i\phi}$
3	0	0	$\psi_{300} = \frac{1}{81\sqrt{3}\pi} \left(\frac{Z}{a_0}\right)^{3/2} \left(27 - 18\frac{Zr}{a_0} + 2\frac{Z^2 r^2}{a_0^2}\right) e^{-Zr/3a_0}$
3	1	0	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(6 - \frac{Zr}{a_0}\right) \frac{Zr}{a_0} e^{-Zr/3a_0} \cos \theta$
3	1	± 1	$\psi_{31\pm 1} = \frac{1}{81\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(6 - \frac{Zr}{a_0}\right) \frac{Zr}{a_0} e^{-Zr/3a_0} \sin \theta e^{\pm i\phi}$
3	2	0	$\psi_{320} = \frac{1}{81\sqrt{6}\pi} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} (3 \cos^2 \theta - 1)$
3	2	± 1	$\psi_{32\pm 1} = \frac{1}{81\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} \sin \theta \cos \theta e^{\pm i\phi}$
3	2	± 2	$\psi_{32\pm 2} = \frac{1}{162\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} \sin^2 \theta e^{\pm i\phi}$

Densità di probabilità radiale



6.3

The eigenvalues and eigenfunctions of L^2 and L_z 6.2 Polar plots of the probability distributions $|Y_{lm}(\theta, \phi)|^2 = (2\pi)^{-1}|\Theta_{lm}(\theta)|^2$.

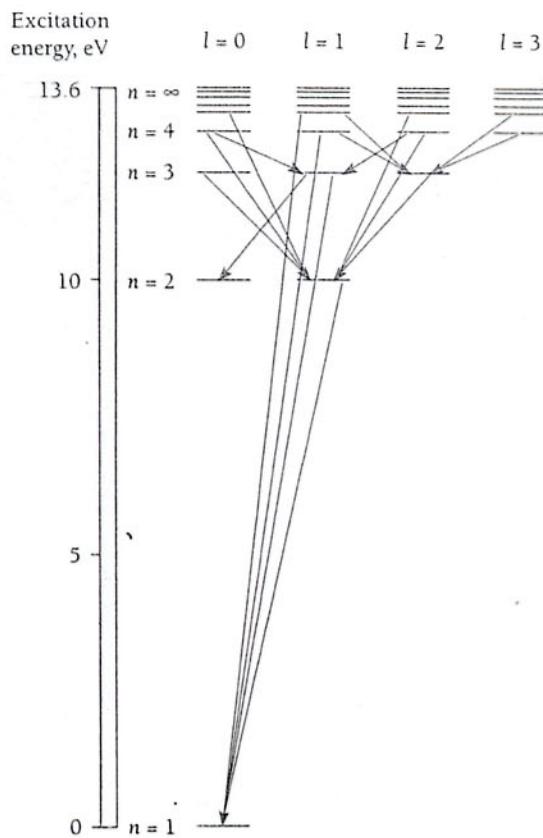
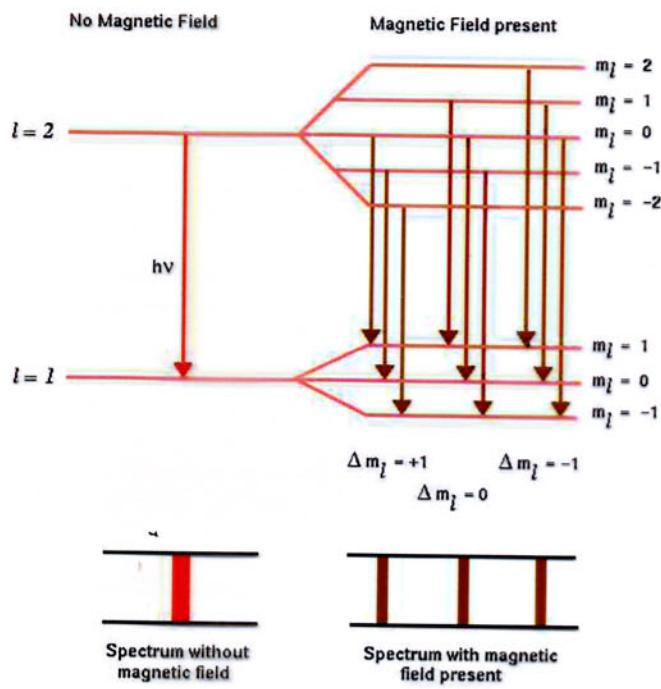


Figure 6.13 Energy-level diagram for hydrogen showing transitions allowed by the selection rule $\Delta l = \pm 1$. In this diagram the vertical axis represents excitation energy above the ground state.



Fisica 5F AA 2002/03

L'effetto Zeeman



La separazione dei livelli energetici:

$$\Delta E = m_l (\epsilon \hbar / 2m) \cdot B$$

Il magnetone di Bohr

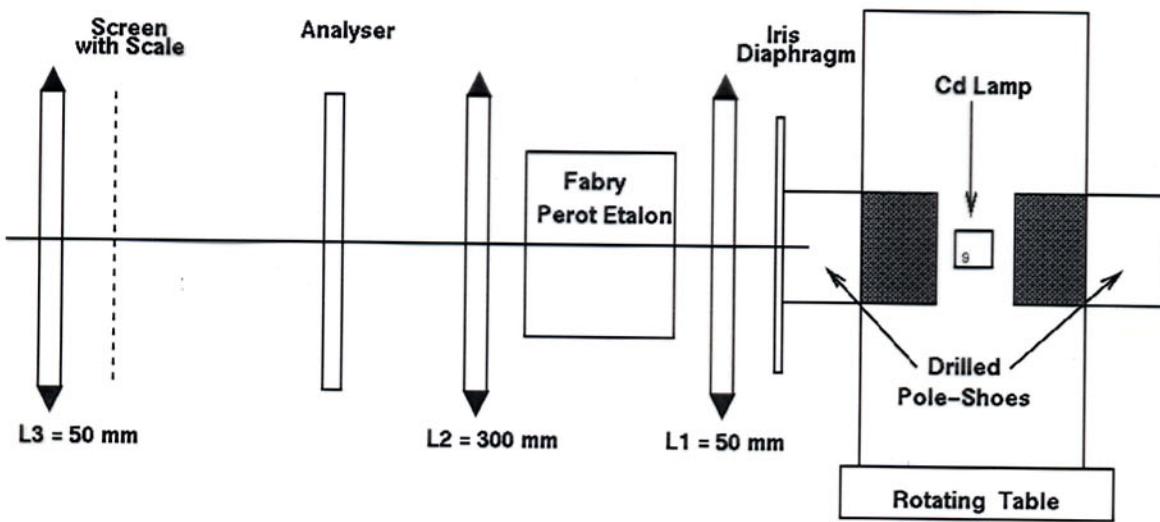
$$\alpha_B = \epsilon \hbar / 2m = 5.79 \cdot 10^{-5} \text{ eV/T}$$

La separazione delle righe dello spettro

$$v = v_0 - (e / 4\pi m) \cdot B$$

$$v = v_0$$

$$v = v_0 + (e / 4\pi m) \cdot B$$

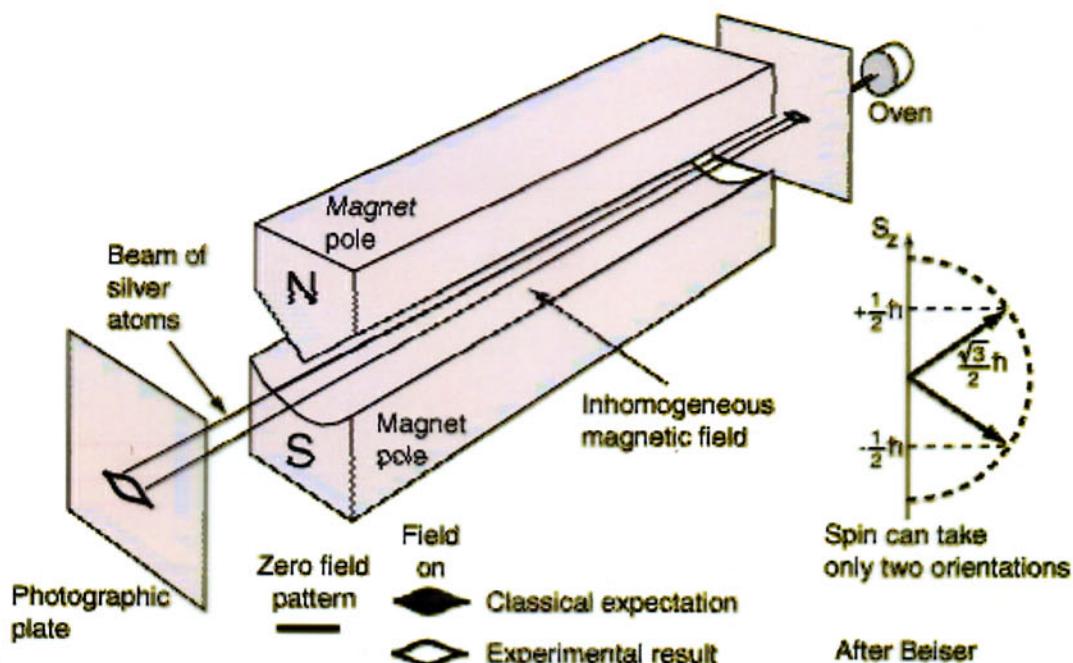




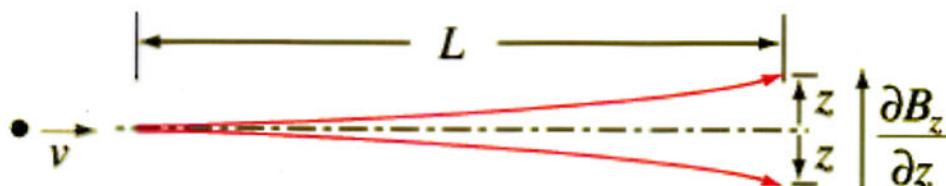
Fisica 5F AA 2002/03

L'esperimento di Stern-Gerlach (1922)

L'energia associata ad un dipolo magnetico immerso in un campo magnetico è $U_m = -\infty \cdot B = \pm \alpha_B B$



La forza che un campo magnetico non uniforme produce sul dipolo è $F_z = -dU_m/dz = \pm \alpha_B \cdot dB/dz$

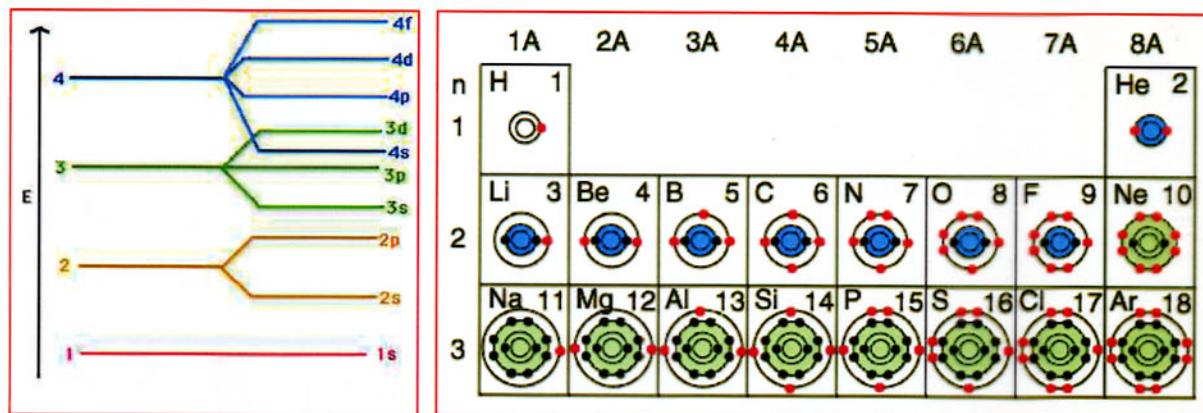


$$z = \frac{1}{2}at^2 = \frac{1}{2} \frac{F}{m} \left[\frac{L}{v} \right]^2 = \pm \frac{\mu_B L^2}{4KE} \frac{\partial B_z}{\partial z}$$



Fisica 5F AA 2002/03

L'occupazione dei livelli energetici in atomi a molti elettroni



State	Principal quantum number n	Orbital quantum number	Magnetic quantum number	Spin quantum number	Maximum number of electrons
1s	1	0	0	$\pm\frac{1}{2}$	2
2s	2	0	0	$\pm\frac{1}{2}$	2
2p	2	1	-1,0,+1	$\pm\frac{1}{2}$	6
3s	3	0	0	$\pm\frac{1}{2}$	2
3p	3	1	-1,0,+1	$\pm\frac{1}{2}$	6
3d	3	2	-2,-1,0,1,2	$\pm\frac{1}{2}$	10