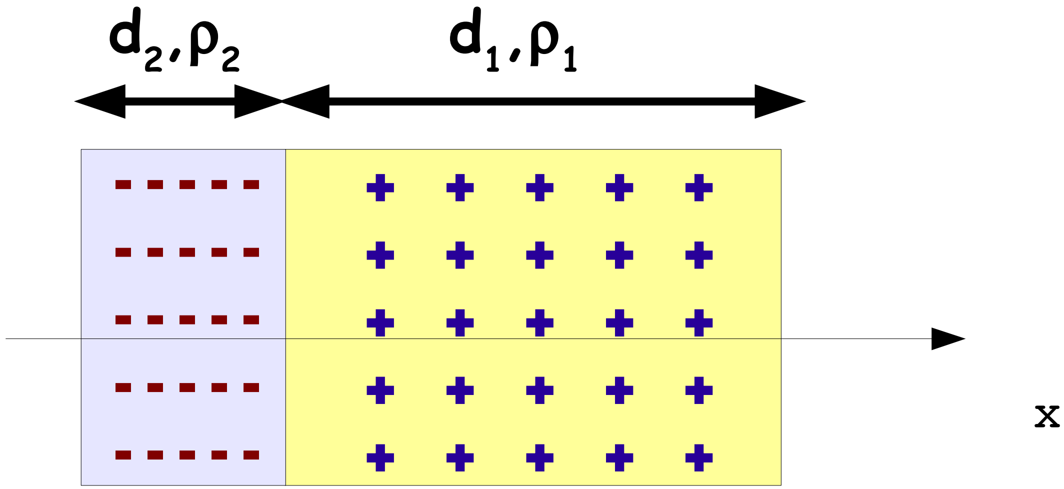
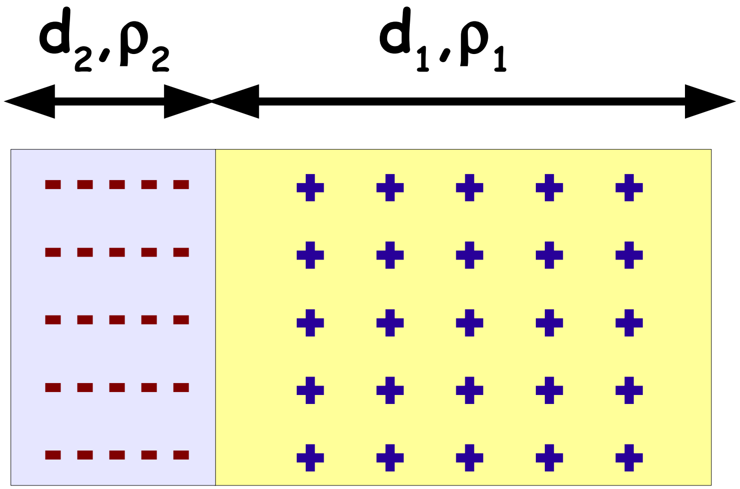


Il sistema in figura consiste di due regioni dello spazio in cui e' depositata uniformemente della carica elettrica con densita' ρ_1= +0.1 C/m^3 (= 6 10^{17} protoni /m^3) e ρ_2 rispettivamente. Le due zone hanno spessore d_1= 50 μm e d_2 = 10 μm (le altre dimensioni sono molto maggiori). Calcolare:

1) Il valore di ρ_2
2) L'espressione del campo elettrico E(x) e del potenziale V(x)
3) la differenza di potenziale tra i due estremi del sistema

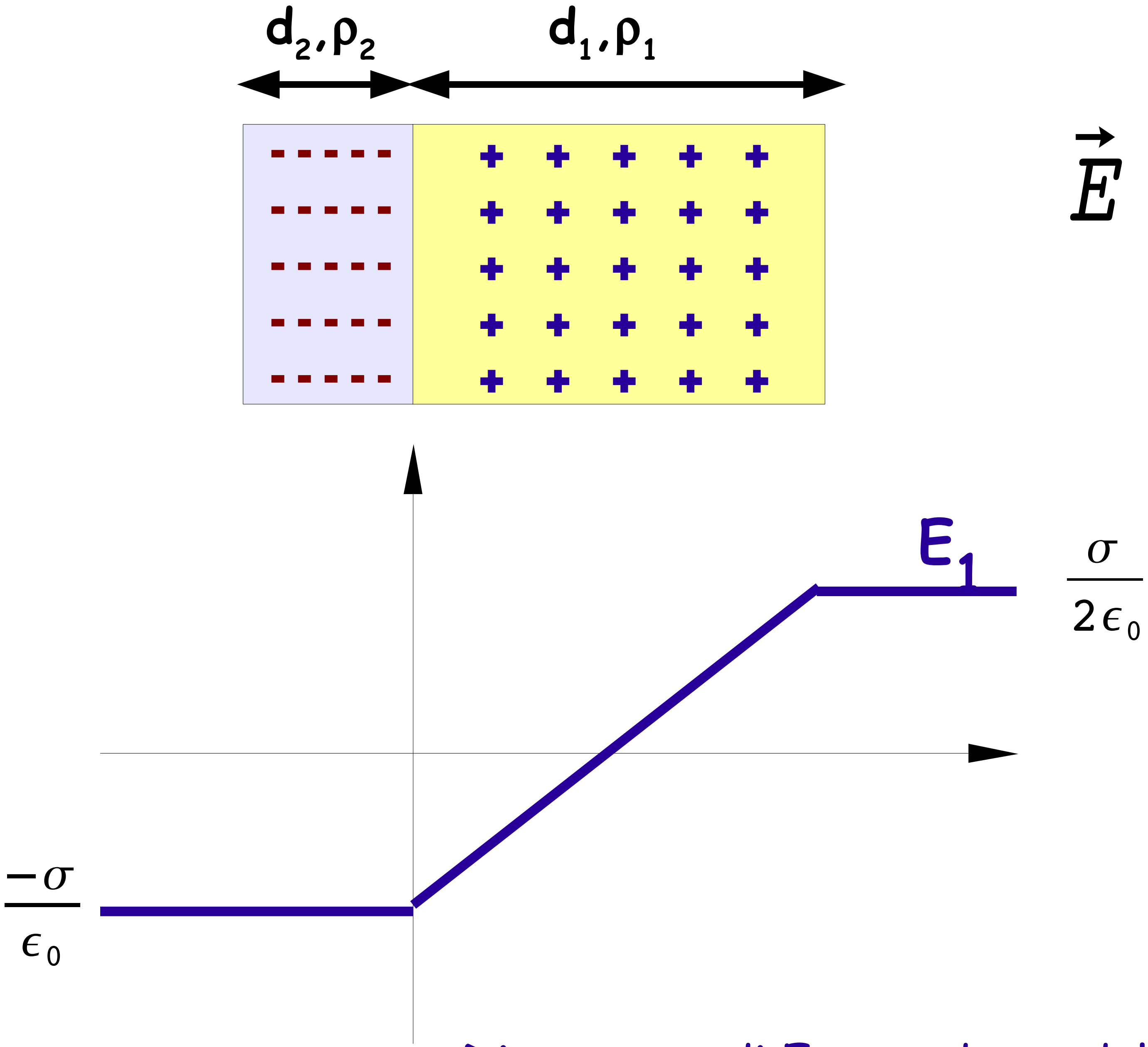


Sistema Neutro :

$$\rho_1 d_1 = -\rho_2 d_2$$

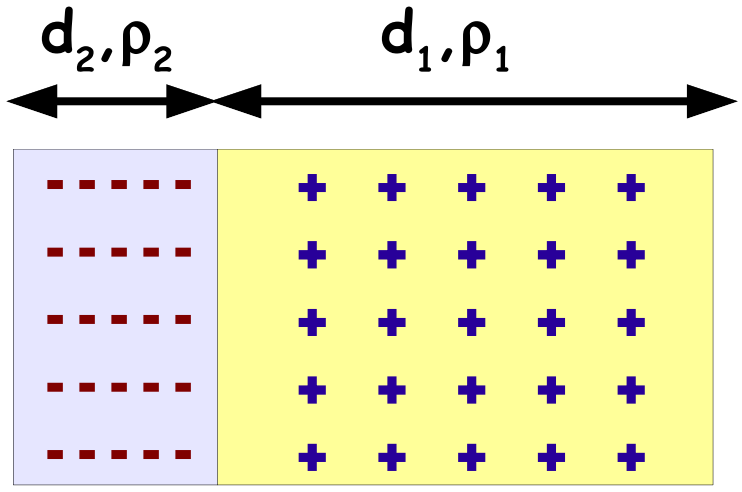
$$\rho_2 = -\rho_1 \frac{d_1}{d_2} = -0.5\cdot 10^{-17} C/m^3$$

$$\sigma = \rho_1 d_1 = -\rho_2 d_2 = 5\cdot 10^{-6} C/m^2$$

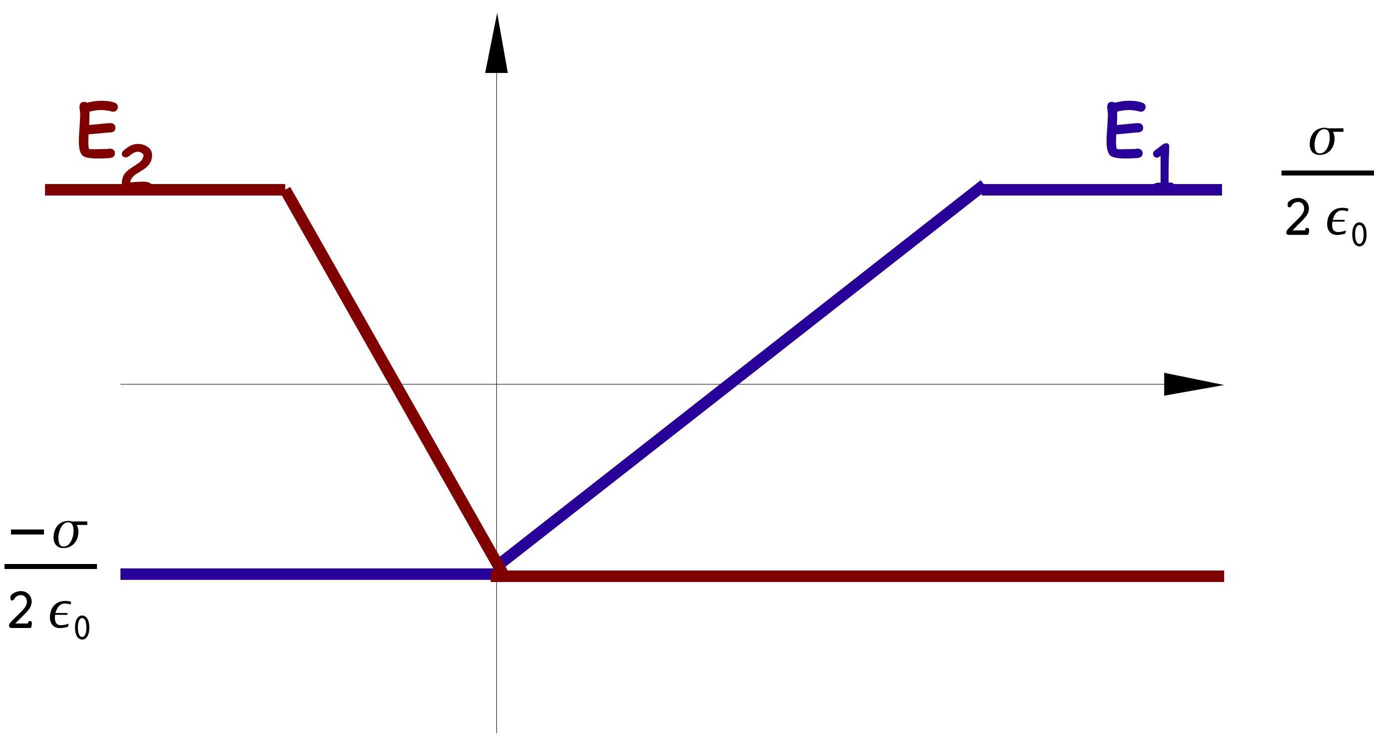


$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

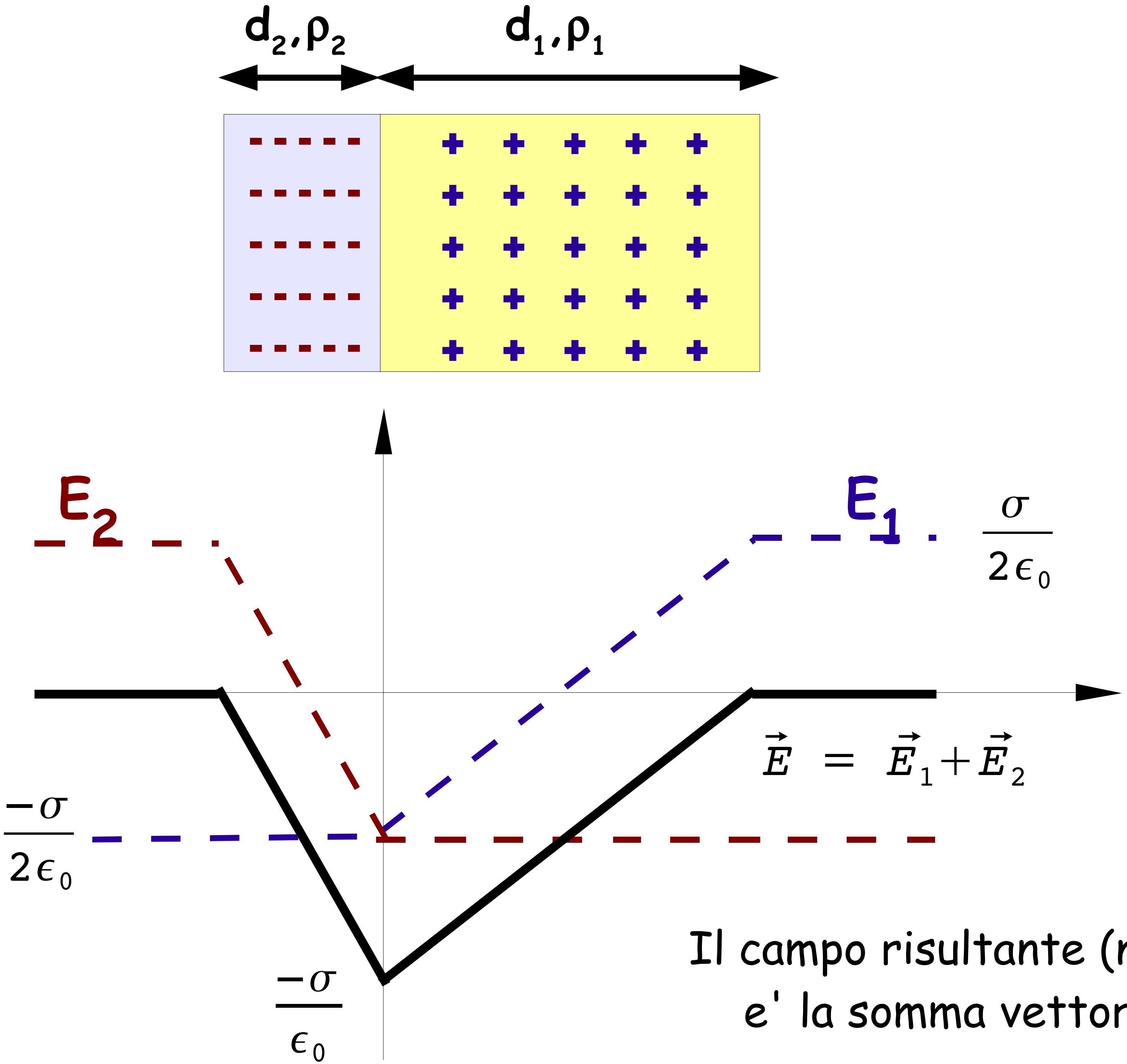
Diagramma di E_1, prodotto dalle sole cariche postive



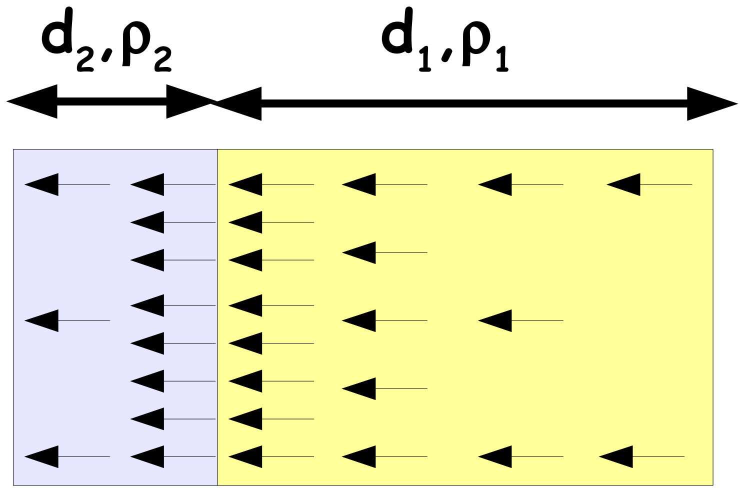
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$



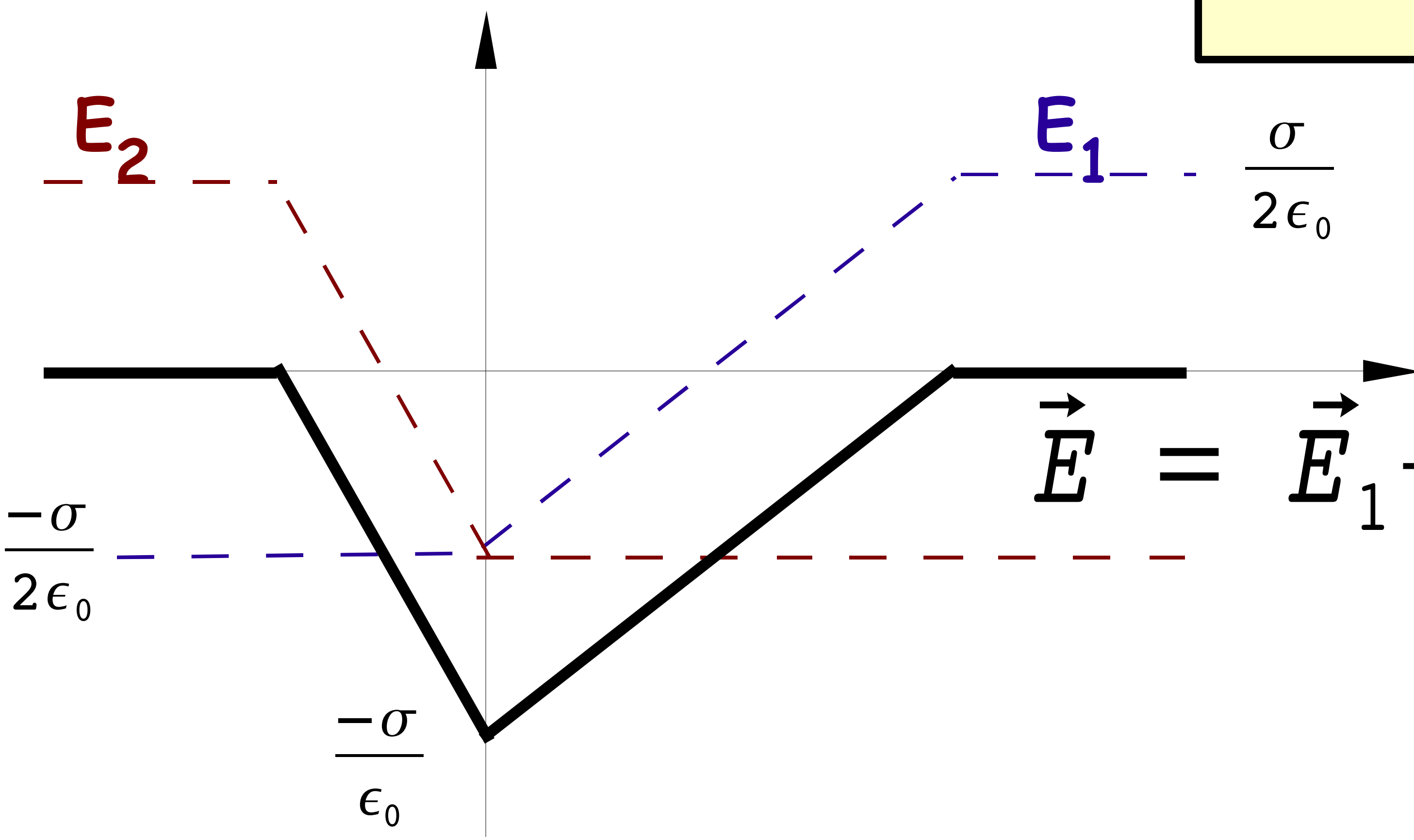
In rosso il contributo E_2 delle cariche negative



Il campo risultante (nero, continuo)
e' la somma vettoriale dei due



$$\vec{E}(x) = \begin{cases} 0 & x > d_1 \\ \dfrac{-\sigma}{\epsilon_0}\left(1 - \dfrac{x}{d_1}\right)\vec{u}_x & 0 < x < d_1 \\ \dfrac{-\sigma}{\epsilon_0}\left(1 + \dfrac{x}{d_2}\right)\vec{u}_x & -d_2 < x < 0 \\ 0 & x < -d_2 \end{cases}$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Posto, ad esempio $V(0) \equiv 0$

Si ha in **$-d_2 < x < 0$**

$$V(x) - V(0) = -\int_0^x E(t)dt = \frac{\sigma}{\epsilon_0}\int_0^x \left(1 + \frac{t}{d_2}\right)dt$$

$$V(x) = \frac{\sigma}{2d_2\epsilon_0}\left[(d_2 + x)^2 - d_2^2\right]$$

Analogamente in **$0 < x < d_1$**

$$V(x) - V(0) = -\int_0^x E(t)dt = \frac{\sigma}{\epsilon_0}\int_0^x \left(1 - \frac{t}{d_1}\right)dt$$

$$V(x) = -\frac{\sigma}{2d_1\epsilon_0}\left[(d_1 - x)^2 - d_1^2\right]$$

Posto, ad esempio $V(0) \equiv 0$

Si ha in $-d_2 < x < 0$

$$V(x) - V(0) = -\int_0^x E(t)\mathrm{d}t = \frac{\sigma}{\epsilon_0}\int_0^x (1 + \frac{t}{d_2})\mathrm{d}t$$

$$V(x) = \frac{\sigma}{2 d_2 \epsilon_0}[(d_2 + x)^2 - d_2^2]$$

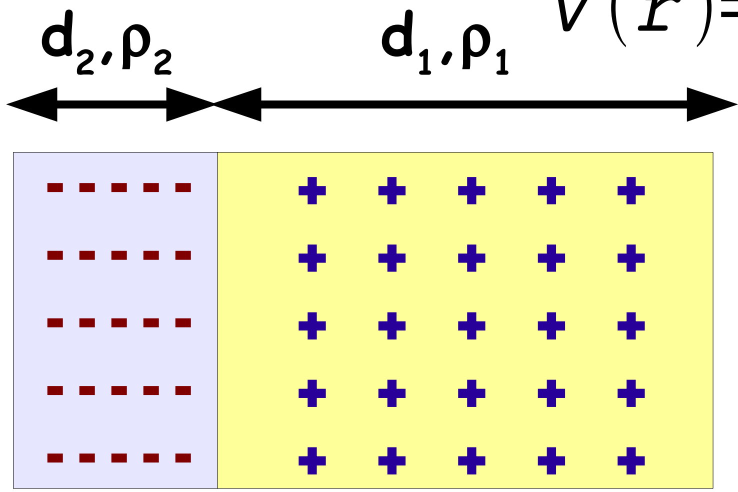
$$V(x \le -d_2) = -\frac{\sigma}{2\epsilon_0} d_2$$

Analogamente in $0 < x < d_1$

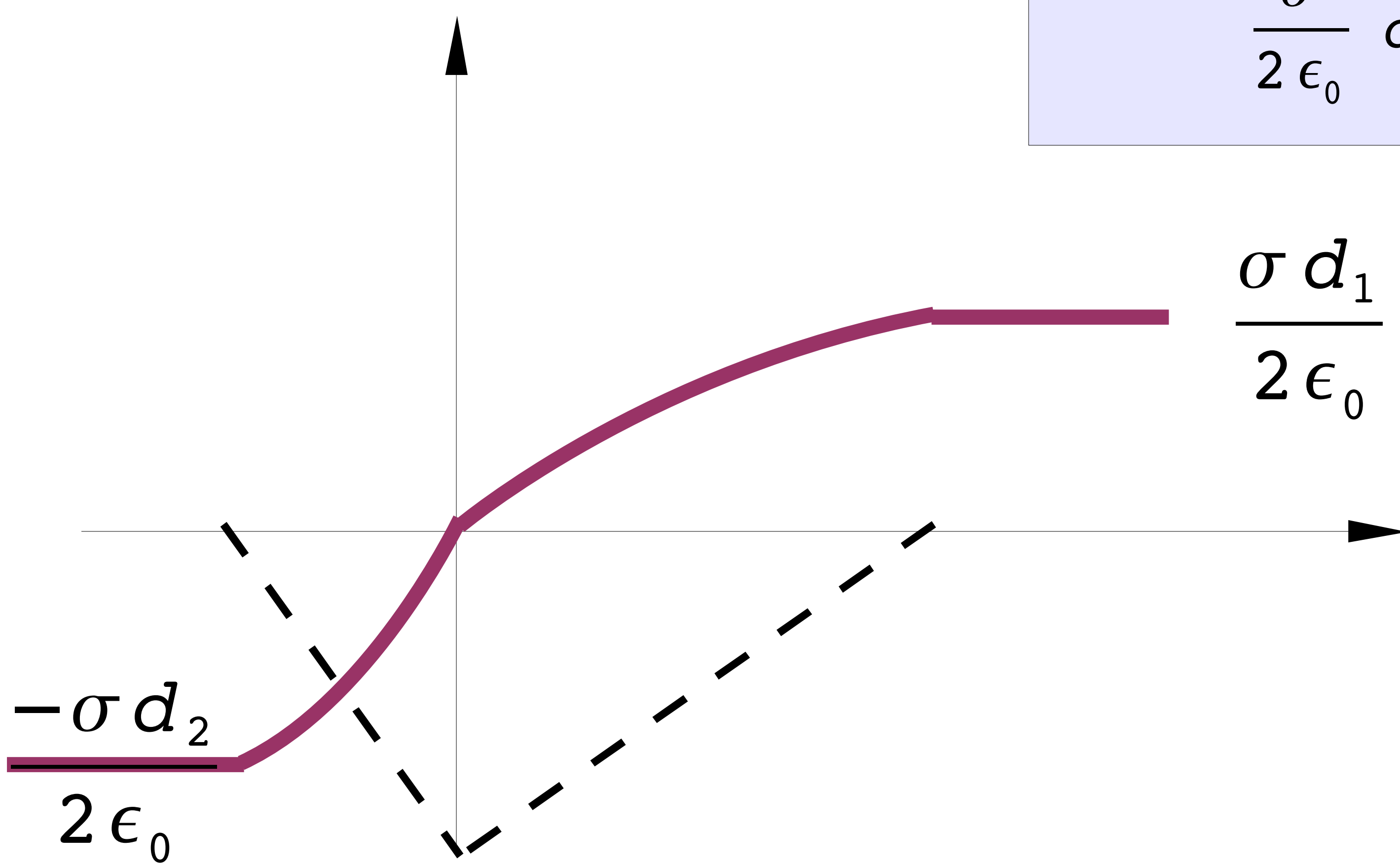
$$V(x) - V(0) = -\int_0^x E(t)dt = \frac{\sigma}{\epsilon_0}\int_0^x (1 - \frac{t}{d_1})dt$$

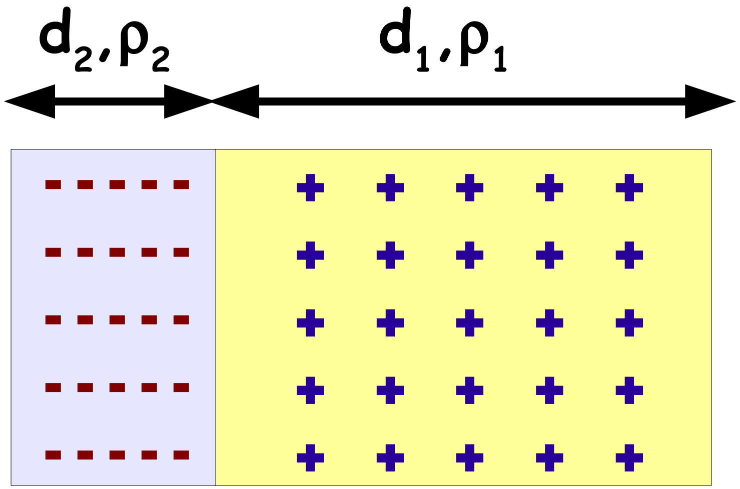
$$V(x) = -\frac{\sigma}{2 d_1 \epsilon_0}[(d_1 - x)^2 - d_1^2]$$

$$V(x \ge d_1) = \frac{\sigma}{2\epsilon_0} d_1$$



$$V(r)=\begin{cases} -\dfrac{\sigma}{2\epsilon_0}d_2 & x\leq -d_2 \\ \dfrac{\sigma}{2d_2\epsilon_0}\left[(d_2+x)^2-d_2^2\right] & -d_2<x<0 \\ \dfrac{-\sigma}{2d_1\epsilon_0}\left[(d_1-x)^2-d_1^2\right] & 0<x<d_1 \\ \dfrac{\sigma}{2\epsilon_0}\,d_1 & x>d_1 \end{cases}$$





$$V(x) = \begin{cases} \dfrac{-\sigma}{2\epsilon_0} d_2 & x \leq -d_2 \\ \dfrac{\sigma}{2 d_2 \epsilon_0} \left[(d_2 + x)^2 - d_2^2 \right] & -d_2 < x < 0 \\ \dfrac{\sigma}{2 d_1 \epsilon_0} \left[d_1^2 - (d_1 - x)^2 \right] & 0 < x < d_1 \\ \dfrac{\sigma}{2\epsilon_0} d_1 & x > d_1 \end{cases}$$

Pertanto la differenza di potenziale e':

$$V(d_1) - V(d_2) = \frac{\sigma}{2\epsilon_0}(d_1 + d_2) = 16.9\, V$$